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PREDETERMINATION OF SPARKING IN DIRECT-CURRENT MACHINES.

BY W. L. WATERS.

Speaking generally, dynamo design did not become an art until after the old two-pole smooth core Siemens and Edison machines came into extensive use for lighting. The original design of these machines was more or less guesswork; but after a few machines had been made to run satisfactorily the designer was able to lay out a complete line of machines, designing them partly by eye and partly by some empirical rules, which he decided on as he built successive machines. The armatures were designed more from a mechanical than from an electrical standpoint, the length being limited by the stiffness of the shaft rather than by questions of commutation. The armatures were not ventilated and the output was necessarily limited by heating. The armatures being of the smooth-core type, the self-induction of the armature coils was so small, even with the length of armatures in general use, that usually it was not necessary to consider it in connection with the sparking of the machine. It was, however, generally recognized that if the magnets were too weak the machine was likely to spark, so the length of the air-gap was usually determined by some empirical rule obtained by experiment. When slotted armatures were adopted extensively they were designed along the same lines as smooth-core armatures. These armatures were so badly ventilated that the output was limited by heating to about one half that of a modern armature; but in spite of this it was found necessary to use carbon brushes to obtain good commutation.

To economize in dies several different lengths of armatures were frequently built on the same diameter, and to economize space the armatures were often built smaller in diameter and longer than they otherwise would be. Experience with these different forms of armature made it very evident that a long armature had a greater tendency to spark than a short one; this became especially noticeable as the ventilating of armatures was improved and the output increased, so as to take advantage of the cooler running.

Up to this time a great deal had been written on the theory of commutation in dynamos, but had been ignored by the practical designers who had more faith in experimental results. This bad behavior of long armatures as regards sparking called attention to the theoretical work, and designers began to consider whether or not the self-induction of the commutated coil did not, after all, decide the amount of current the machine would carry without sparking. In the first attempts to take into account the self-induction of the commutated coil, the self-induction of a one-turn coil was considered as being simply proportional to the length of the armature core; that is, the shape and size of the slot, the number of coils per slot, and the self-induction of the end connections—all were neglected. This gave a very simple formula for the self-induction:

$$L = ln^2.$$

Where l = length of armature
n = number of turns per coil,

And the self-induction e.m.f. of commutation (the reactance voltage as it was called) which is an estimate of the difficulty of commutating the current, was given by

$$E = l\,n^2\,I\,f$$

I being the current per coil and f the frequency of commutation.

This formula gave good results when applied to machines designed along the same general lines. The allowable value of the reactance voltage could be obtained from experiment on one machine and used in the design of other machines. But, if applied to machines which were designed differently, the formula showed wide discrepancies; so it soon became recognized that the formula was at best only a rough approximation.

Early slotted machines were designed with one coil per slot; two coils per slot obviously saved insulation space and was soon tried, but it was found that generally every other bar on the commutator became badly marked.

As it was imperative to save space in car motors, three coils per slot were adopted, and in extreme cases four, or even five coils per slot were used. It was generally found, however, that whenever more than one coil per slot was used some of the commutator-bars were marked, and that it was possible to count the number of coils per slot by the recurrence of the marking on the commutator. This marking was attributed to the inequality caused by using a small number of slots, and so the general rules were adopted—to use as many slots as possible and to make small machines with one coil per slot and large machines with two coils per slot.

It was also noticed that the dead coils necessary in certain multipolar wave-windings often caused some of the commutator-bars to be marked; this was naturally attributed to the dissymetry produced in the winding, and it became generally recognized that anything tending to produce inequality in the commutation conditions, such as few slots or many coils per slot, or dead coils, tended to make perfect commutation more difficult.

With increased competition came the necessity of cheapening the cost of building these machines; designers then returned to the construction of several coils per slot. In reducing the amount of copper on the armatures to save in the cost of material, it naturally happened that shallow slots were used. And it was found that with these wide and shallow slots it was possible to obtain good commutation with several coils per slot, under conditions where it would be quite impossible with the old deep and narrow slots. Obviously this was due to the lesser self-induction of a wide slot compared to a narrow one, and it was soon acknowledged that the shape of the slot should be considered in calculating the self-induction of the commutated coil.

When designing an armature for small self-induction it would be natural to make it large in diameter and short in length; that is, with a large pole-pitch. But in carrying this to an extreme it was found that it did not give the good results expected. It was suggested that this result was due to the fact that the self-induction of the end connections had been neglected and that in armatures with large pole-pitch and short length of core, the self-induction of the end connections was comparable with that of the conductor embedded in the slots.

In the light of these experiences it is evident that the design of a direct-current machine in regard to sparking is a compromise between a number of conflicting conditions. It is not possible to obtain a formula which will give a strict measure of the commutating qualities of all machines; but by taking into consideration the more important conditions which effect the sparking it is possible to obtain a formula which will give fairly accurate results when applied to machines similarly designed, and which will give some idea of the sparking when applied to machines of widely different design. Such a formula, when it has been applied to a number of machines of different types, so that the allowable values for the sparking constant have been determined, can be taken as a fair working formula, and can be placed in the same category as empirical formulas for determining the regulation of alternators. Such formulas are not intended to reduce the designing of machines to mere slide-rule work, but are intended simply to give an idea as to the experimental results to be expected from individual design.

As outlined above, the most important conditions to be taken into consideration are the self-induction pressure of the commutated coil and the inequalities introduced by the conditions of commutation.

I.—Self-Induction Pressure.

This is given by the formula; V = Self-induction of one coil times number of coils commutated in series times current in coil times frequency of commutation.

The self-induction of one coil = (self-induction of one conductor embedded in the slot + self-induction of one end connection) times (number of turns per coil)^2.

The self-induction of one conductor embedded in the slot = $l\,k$. Where l is the length of the core and k is a constant depending on the dimensions of the slot.

By determining the self-induction of a large number of slots we find that this constant k can with sufficient accuracy be taken as a function of the ratio

$$r = \frac{\text{Width of slot.}}{\text{Depth of slot.}}$$

A curve can be plotted connecting r and k, determined experimentally from tests on a number of armatures; such a curve is shown in Fig. 1.

Take the self-induction of the end connections = length of end connections times constant c'. And as the length of end connection is approximately proportional to the pole-pitch this can be written with sufficient accuracy = $p\,c$.

Hence the self-induction of one coil $= n^2 (l\,k + p\,c)$.

The number of coils commutated in series, N, is, of course, one in a parallel or lap-wound armature, and equal to the number of pairs of poles in a series or wave-wound armature.

The current per coil I in a series or wave-wound armature is equal to one half the total current in the machine, while in a parallel or lap-wound armature it equals the total current divided by the number of poles.

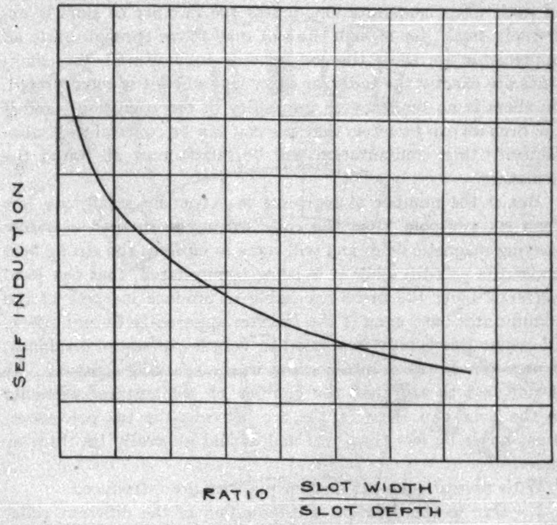

FIG. 1.

The self-induction pressure of the commutated coil is then given by

$$V = n^2 (l\ k + p\ c)\,N\ I\ f$$

Where f = frequency of commutation = number of commutator-bars times speed in rev. per min.

The width of the brush is neglected in calculating the frequency of commutation, since it is found by experiment that within the ordinary limits of practice the thickness of the brush has no effect on the operation of a machine, unless the current density is excessive. The probable explanation of this is that a thicker brush gives more time for commutation to take place; but it means that more coils are commutated at the same time, thus increasing the self-induction effect. These two effects apparently counterbalance each other to a great extent.

II.—INEQUALITIES DUE TO CONDITIONS OF COMMUTATION.

These are due to the use of a few slots; more than one coil per slot; and to dead coils.

If there is only one coil per slot the use of few slots does not in itself affect commutation, unless the number of slots is extremely small; for though the slot may move through quite an appreciable arc while the coil is being commutated, the conditions are exactly the same for every coil when it is commutated. So there is no tendency to inequality in the conditions, and if the brushes can be set so that one coil can be commutated satisfactorily then commutation will be satisfactory all round the armature.

But if the number of segments is extremely small, say less than six per pole, then the coils will move in such a widely varying magnetic field, and will come so close to the strong field under the pole-tip while it is being commutated, that the local currents under the brush are liable to produce marking of the commutator-bars even if the brushes apparently do not spark. Of course this is only important in very low-pressure machines; it need not be taken into account in the sparking constant. It is sufficient to say that the number of commutator-segments in the polar-gap; that is, the arc between the two pole-shoes, must never be less than two and should generally be three or more.

With several coils per slot inequalities are introduced:

1.—Due to the different self-induction of the different coils;

2.—Due to commutation under different conditions.

The self-induction of all the armature coils will be the same when there are only two coils per slot, as it is obvious that the configuration of the conductors and neighboring iron is the same for both coils. But when there are three or more coils per slot the self-induction of the various coils will vary, as they occupy different relative positions in regard to the iron; the self-induction of the center coil will be less than the self-induction of the outer coil. Investigating conditions at the point of commutation in a modern generator by means of a pilot-brush, it is found that in practically all cases commutation goes on at a point where there is no resultant magnetic field; that is, at a point where the armature field just counterbalances the field due to the magnets. In other words, there is resistance commutation; the armature current is commutated by the varying resistance of the brush, rather than by a reversing e.m.f. due to passing through a magnetic field. This being the case there is need to consider only the self-induction of those coils which have the greatest self-induction. If these are commutated satisfactorily by means of the varying resistance of the brushes, then those coils which have a smaller self-induction will also be commutated satisfactorily. Hence the variation in the self-induction of the coils

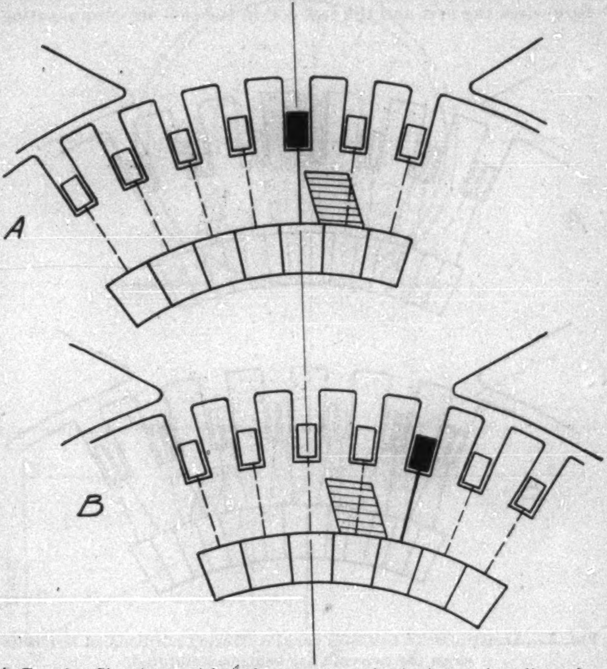

FIG. 2.—Showing position of armature at beginning and ending of commutation period.

need not be considered and in the formula all that needs to be considered is the self-induction of those coils which have the greatest self-induction.

The chief inequality introduced into the commutation by having more than one coil per slot is due to the various coils in the slot commutating when they are in different magnetic fields. This is evident from Fig. 3, which shows the position of the armature when the first and the last coil in the slot are commutating.

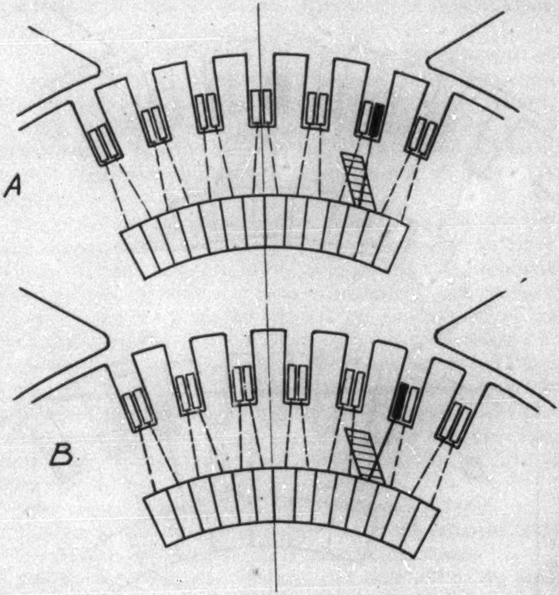

FIG. 3.—Armature with two coils per slot showing positions of armature when the two coils are being commutated.

If the brushes are set so that the magnetic field is right for the first coil it will be wrong for the last one, and vice versa. So whenever the machine is loaded to its limit the commutating conditions will be so bad for some of the coils that in time some of the commutator-bars will become pitted and there will be that well-known regularly recurring marking of the commutator-bars.

The question now is how to take this inequality into account in the sparking formula. To do this, make the assumptions that the magnetic field varies uniformly from the neutral point to the pole-tip, and that in order to obtain perfect commutation

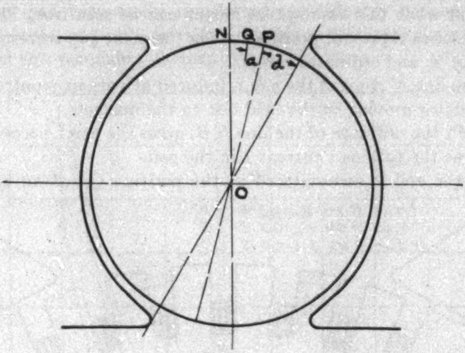

FIG. 4.

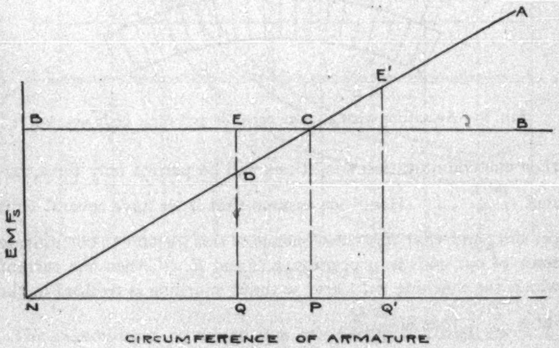

FIG. 5.

it is necessary to move the brushes from a position at the neutral point at no load to a position half-way between the neutral point and the pole-tip at full load. Calling the distance between the neutral point and the pole-tip $2\,d$, and assuming the brushes fixed on the line $O\,P$ half-way between the pole-tip and the neutral point, then if any coil is commutated when it is at Q distant " a " from P, then it will only commutate perfectly a load $l - \frac{a}{d}$.

Just what this assumption means can be seen from Fig. 5.

Abscissa represent positions along the polar-gap corresponding to Fig. 4, and ordinates e.m.fs.

The line $N\,A$ gives the e.m.f. induced at various points by the conductor moving in the field due to the magnets.

$C\,P$, the ordinate of the line $B\,B$, gives the e.m.f. necessary to reverse the full-load current I in the coil.

If the coil is commutated at the position Q instead of at P then the commutation conditions will be perfect only for a current $\frac{D\,Q}{C\,P} \cdot I$. Hence we assume that if we have several coils per slot, and that if in consequence of this we have to commutate some of our coils in a position $E\,Q$ and $E'\,Q'$, then the current which the machine will carry without sparking is reduced in the ratio $\frac{D\,Q}{C\,P}$ that is $\frac{N\,Q}{N\,P}$.

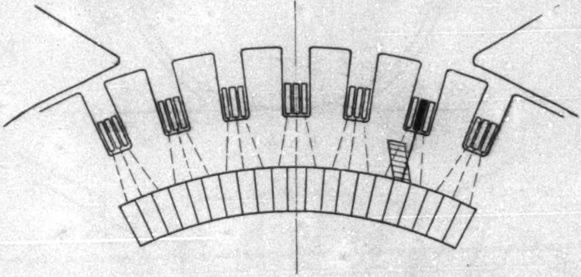

FIG. 6.—Armature with 20 slots per pole and three coils per slot.

It is very easy to figure out just what this inequality amounts to in any particular case. Take 20 slots per pole, 3 coils per slot, pole-face = 75% of pole-pitch.

There are 2.5 slots between the neutral point and the pole-tip. Assuming that the conditions are perfect for the center coil, the outer coils are 0.333-slot pitch out of this most favorable position.

And 1.25 slots corresponding to variation from no load to full load, hence an equality of 0.333-slot pitch gives an inequality factor $\frac{0.333}{1.25} = 0.26$. Hence the sparking constant should be multiplied by the inequality factor 1.26.

Curves can very easily be plotted for different numbers of slots per pole, and coils per slot, in order to facilitate the calculation of this inequality factor.

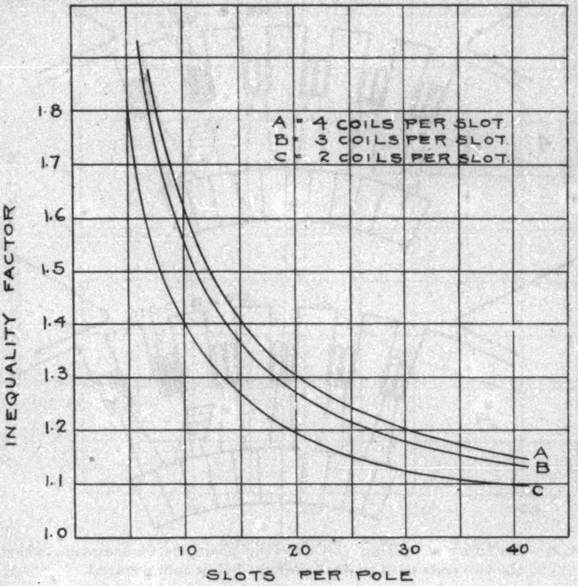

FIG. 7.

The assumptions on which this calculation is based are to a great extent rational, and though we cannot pretend that the calculation has a rigid basis, yet it is probably as correct as the other sparking calculations, and used with discretion it gives fairly reliable results.

The inequality introduced by using a dead coil on the armature is similar to that due to several coils per slot. The dead coil produces a break in the uniformity of the winding; and if the position of the brush is correct for commutation of the coil immediately on one side of the dead coil then it will be just one segment out of the correct position for the coil immediately on the other side of the dead coil. The inequality introduced can be calculated, and allowed for in the sparking constant exactly in the same way as we allow for the inequality due to

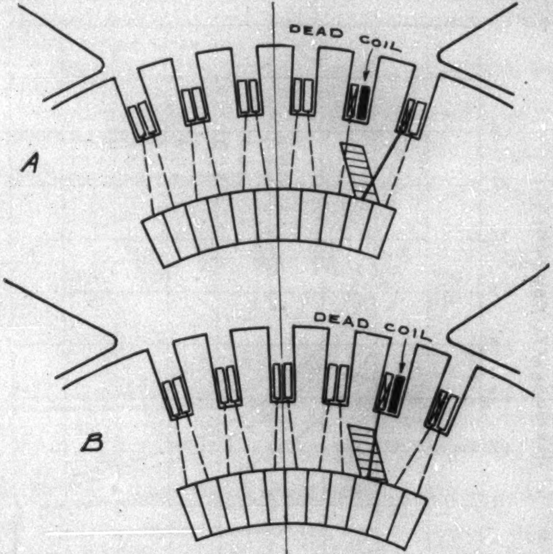

Fig. 8.—Armature with dead coil showing positions of armature when the two coils next to dead coil are being commutated.

several coils per slot. Assuming that the brush is in a mean position then it will be just half a segment out of position for the two coils which are next in position to the commutated coil.

Making the same assumptions as before; if there are n commutator-segments per pole, and if the pole face = 75% of pole-pitch then the inequality is equivalent to $\frac{8}{n}$. Thus if there are 20 segments per pole a dead coil produces an inequality equal to 40% of the load, and the inequality should be introduced into the sparking constant by the factor 1.4. A curve can readily be plotted between the inequality factor and the number of coils per pole. Such a curve is shown in Fig. 9.

Combining all the different factors which affect sparking we get our complete formula for a sparking constant.

$$C = n^2 \ (l k + p c) N \ I \ f \ P \ Q.$$

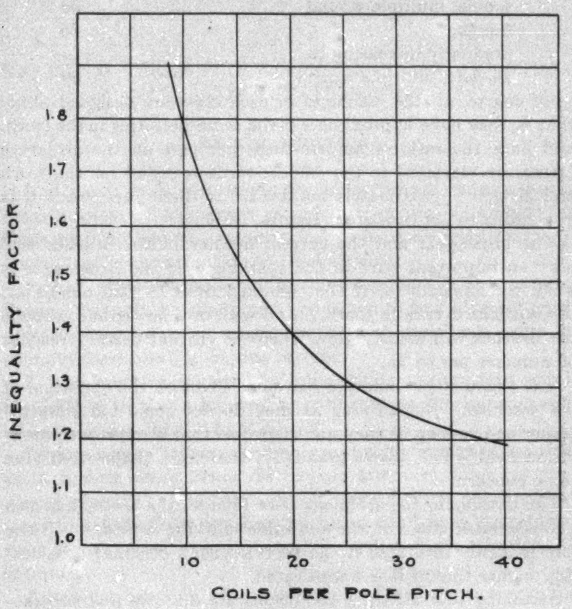

FIG. 9.

P being the inequality factor resulting from a number of coils per slot.

Q being the inequality factor resulting from a dead coil.

This formula is not put forward as being scientifically exact, but as an empirical formula which has gradually been built up as the result of experience, different terms having been added to the formula from time to time as it was found necessary to take different conditions into account. As the formula stands it gives excellent results. When we know the value of C we can allow for the particular design of machine we are dealing with.

The relative values of C that have been found allowable in different cases are somewhat as follows:

2-pole	20
4-pole, series wound	35
6-pole, " "	50
4-pole, multiple wound	30
6-pole, " "	35
gradually increasing to	
24-pole, multiple wound	50

Of course, all the machines in each class are designed alike; that is, they have approximately the same densities in the teeth, and have the ratio of ampere-turns per pole on the armature (armature reaction) to the ampere-turns required for the teeth and air-gap approximately constant. If these vary much it is impossible to get consistent results.

The brush-gear and the current density in the brushes also play an important part in the sparking. If the brush-gear is weak mechanically, or if the commutator is in bad condition, the machine is sure to spark. And with ordinary-grade carbons the brushes will usually glow when the current density reaches 50 amperes per sq. in.

The shape of the pole-tips has some effect on the operation of the machine. But so long as they do not come too close together and so long as they are shaped so that there is a commutation field which varies gradually, the exact shape need give us no concern.

The density in the armature core (behind the teeth) has also a considerable effect on the allowable sparking constant. If the core is highly saturated it can be run with a constant considerably higher than if it is unsaturated.

Assuming that all these conditions are uniform and satisfactory, the variation in the allowable value of C shows that the formula does not take into account all the conditions that affect the sparking, so the formula must be used with considerable discretion. It cannot be claimed that the formula is in any way accurate, but it certainly is an empirical working formula, capable of giving very good results when carefully used, and as such it is put forward.

A paper to be presented at the 187th Meeting of the American Institute of Electrical Engineers, New York, May 17–18, 1904.

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EFFECT OF SELF-INDUCTION ON RAILWAY MOTOR COMMUTATION.

BY E. H. ANDERSON.

This paper deals with fundamental features of railway motor commutation. Oscillograph records are employed to illustrate these features and no mathematical deductions are attempted. I would call your attention to the following:

1. The pressure produced by interrupting the line current flowing through the conductors connected to two adjacent commutator-bars, or what might be termed reactance volts of commutation based upon line current.

2. The pressure produced by interrupting local current which is additive to that produced by interrupting the line current, and the causes for the local current.

Direct-current railway motors are usually designed for 500 volts, having series drum distributed armature windings connected directly to the commutator; have definite polar projections and field coils, the brushes being fixed in a mechanical neutral and suited to operate equally well in both directions of rotation.

Reactance pressure of commutation of line current may be calculated directly as any ordinary reactance, depending upon the shape of slot, length of iron in armature body, speed, kilowatts, armature diameter, frequency, etc. From any general expression for this reactance volts, there will appear in the numerator kilowatts and in the denominator amperes; which is to say that the greater the amperes, the less the reactance volts—at first seemingly absurd, but when it is considered that all other values are constant the expression means that the greater the amperes the less the pressure, and that the reactance pressure of commutation of line current decreases as the pressure of the motor; in brief, this means that a motor of identically the same design will commutate better at 250 than at 500 volts. This is a matter of everyday personal experience, and needs no further discussion.

The point brought out in the above is that high-pressure machines are difficult to build; and good commutation with reasonable cost is unfortunately limited to relatively low working pressures, low compared with the power involved.

Taking the frequency of commutation as the relation of the number of commutator-bars to the width of brush, a low value of reactance volts is calculated. For good commutating machines this value is a fraction of a volt. Reactance pressure between the commutator and the brush as indicated by the spark is evidently much higher, and oscillograms hereafter given show this to be so. Fig. 1 appears to me to explain this. Estimating on a uniform change of current throughout the width of the brush, we get a maximum value represented by *C D*. Consider that for the first half of the brush the line current is being increased by the addition of local current, then the pressure across the first half of the brush will be negative; as the commutator bars proceed across the brush the sign of the voltage reverses. In the latter part of the brush the current

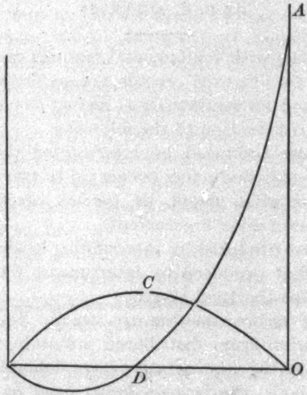


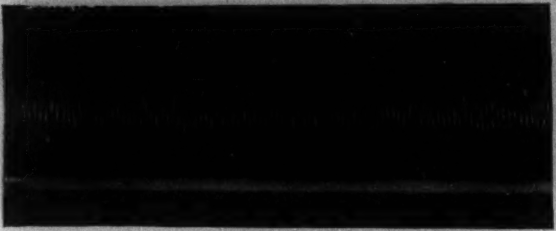
FIG. 1.

decreases, the change in current becoming more and more rapid as the bar nears the edge of the brush, until at the point of rupture the change of current is very great, the frequency thus represented being many times that of the frequency indicated by the number of commutator-bars to width of brush, consequently we get a value represented by *O A* instead of the value *C D*.

OSCILLOGRAM 1.
General electric railway motor. Pressure between two exploring brushes 7-32 in. apart, one 1-16 in. behind motor brush. 304 amperes in field and armature; 453 volts, 482 rev. per min.

Oscillogram 1 shows the pressure between two exploring brushes, one placed under the brush and the other 1/16 in. ahead of the brush. The maximum values are, as indicated, approximately four volts, and since there are evidently some losses in the contacts of the exploring brushes, the real pressure may be more than indicated. The motor on which this oscillogram was taken is rated on the hourly basis at approximately 300 amperes and 500 volts. The armature is designed with five coils—10 conductors—per slot, and while this may apparently be a radical distribution of conductors, the values in the oscillogram of the pressure of commutation over all bars are approximately the same. This indicates that even with five coils per slot the armature can be so connected as to equalize the commutation for all bars, and still get the advantages of five coils per slot construction.

The pressure produced by interrupting the local current is of course dependent upon the frequency as above outlined in Fig. 1; in fact the whole current, line and local, produces the reactance volts of commutation. This local current is produced by a pressure generated in the conductors connected to the commutator-bars which lie under the brushes. This pressure is due to the shifting of the field caused by the armature magnetizing effect, and is influenced somewhat by the volts per bar of the motor. It will be appreciated that this local current must be cut down as much as possible, since the brushes are always in mechanical neutral, and consequently railway motors have, as a rule, low armature turns and very high flux and field turns compared with those in the usual direct-current generator.



OSCILLOGRAM 2.
General Electric railway motor. Pressure between one brush or motor and an exploring brush. 300 amperes, 520 rev. per min. Run as a motor.

OSCILLOGRAM 3.
General Electric railway motor; 300 amperes in field, 500 rev. per min. Carbon brushes removed. Pressure between commutator-bars, 5-6.

Referring to Oscillogram 2, the minimum pressure between the motor brush and the exploring brush is higher than can be accounted for by the *I R* drop of line current—evidently there is added to the line current enough local current to produce this drop. Since in large motors the local copper circuit is of low resistance compared to the brush contact, nearly all the energy of the local current is delivered as heat on the commutator. Should the motor be very large, the stability of the field relatively to the armature must be great in order to limit the local current to a low value, else the motor will spark and glow at the brushes.

Conversely, on small machines the resistance of the local copper circuit may be high compared to the carbon contact resistance. The same volts induced in the local circuit produced by the distortion of the field do not cause proportionately great local currents to flow, thus greater reactance pressure and greater current density in the brush can be used in design.

Oscillogram 3 is a record of pressure between two commutator-bars with full field excitation, the armature being driven and no current flowing through it, showing the distribution of the flux.

Oscillogram 4.
Pressure between commutator-bars 4-5; 300 amperes, 465 volts, 497 rev. per min.

Oscillogram 5.
Pressure between commutator bars 1-2; 90 amperes, 475 volts, 950 rev. per min.

Oscillogram 4 is the same as Oscillogram 3 except that the armature is loaded and the machine runs as a simple series motor with full rated current. Note that the flux is shifted slightly and the pressure peaks of commutation are shown near the zero line. Several pressure peaks are shown which are evidently caused by the mutual induction, other conductors in the same slots being commutated.

In Oscillograms 3 and 4 two adjacent commutator-bars were connected to collector rings and the record of pressure taken between these.

Oscillogram 5 is taken under the same condition as Oscillogram 4 except that the load on the motor is slightly less than one-third the rating. With one-third excitation the densities are approximately one half, and from Oscillograms 4 and 5 it will be seen that the distortion of the flux at one-third load is much greater than with full load, indicating that the stability of the flux is quite essential to good commutation. Theoretically this stability can be gotten in two ways:

1. The usual method of great saturation.
2. By compensating for the disturbing element (armature reaction).

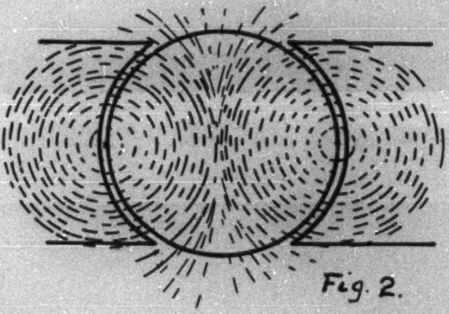


Fig. 2.

Theoretically the latter points to a motor with distributed winding on the field, pole-faces of greater pitch, equal density over the pole-face, relatively few field turns, and the possibility of higher pressure motors.

Fig. 2 shows the effect of armature m.m.f. when there is no main flux.

Oscillogram 6 shows the pressure between two commutator-bars when there is full-load current in the armature and no current in the field, the motor being driven at approximately rated speed. It will be seen that this agrees with Fig. 2. Were the armature completely surrounded with iron pole-face, the peak of the wave would occur where the brushes are located. The position of the brushes is indicated by the several minor peaks of pressure caused by the commutation of the line and the large local current.

Oscillogram 6.
Pressure between commutator bars 5-6. No current in field. 300 amperes in armature, 500 rev. per min.

Oscillogram 7.
General Electric railway motor. No current in field. Pressure between one brush of motor and an exploring brush; 507 amperes in armature, 530 rev. per min.

Oscillogram 7 indicates very forcibly the action of the brushes when running with full current in the armature and no field flux. The pressure between the motor brush and the exploring brush just ahead of the motor brush is at the minimum, six to eight volts, and since only about one volt can be accounted for by the IR of the line current, there is probably a density in the brush contact six to eight times as great. The brushes under test actually get very hot and act badly. It will be seen that there is an average drop of about 10 volts and with 300 amperes line current will give 3 kw. which will account for the heating of the brushes.

Oscillograms 6 and 7 and Fig. 2 will be referred to later where their practical value will be seen.

There is another phase of this subject which is rather new in the electrical world and of considerable importance to engineers and to operators of large direct-current motors. I refer to the proper design of these large direct-current motors so as to eliminate as much as possible the tendency of motors to flash-over when running at free speeds and light currents, the current being interrupted and contact restored again with full pressure across the motors. For some time I have had the opportunity and pleasure of investigating these phenomena for the General Electric Company. The investigation has been continued over considerable time, and a great many tests of almost every character have been made. A few of the fundamental will be illustrated by oscillograms.

Field coils consisting of ribbon copper wound into mica-insulated brass or metallic spools, the last turn of each section riveted to the turn below and forming a band which holds the winding and terminal firmly in place, will, for lack of a better name, be called *standard fields.* We shall also take another construction of field winding where the brass or metallic spool is not used, or approximately the same condition where the spool is cut so as not to form a continuous metallic belt around the pole. The last turn in each section instead of being sweated and riveted to the turn below is insulated therefrom, thus doing away with the copper belt round the pole. Coils constructed in this manner, will, for lack of a better name, be called *special fields*.

It was found that motors with exceedingly good commutation at normal loads and even on overloads at rated pressure, and motors which would commutate without flashing-over with full-load current at double rated voltage, had a peculiar tendency to flash-over when the current was changed rapidly or the circuit interrupted and made again with full pressure on the motors. This phenomenon almost disappeared at heavy loads; that is, at low armature speeds, but increased and became serious at light free-running speeds; that is, high armature speeds with light loads. The flashing pressure sometimes came within the range of high sub-station pressure. These phenomena were reproduced on the stand in various ways and the flashing points determined for a great number of conditions of load, time of interruption, character of contact points, etc. The flashing-over could be accounted for in two ways:

1. That the main flux of the motor changed very rapidly

(when current was applied at full pressure) through the frame; and since the armature coils lying under the brush would encircle the whole of this flux, a great transformer pressure would be induced in these coils causing large local current to flow in the coil and through the brush contact. On interrupting this current as the coil passed out from under the brush, there would be a flash across the mica from bar to bar; and there would be sufficient pressure to maintain it, consequently the line current would follow the local flash from bar to bar and thus flash-over.

2. That the predominating cause was the slowness of the main flux to respond to excitation, allowing a great rush of current. All metallic belts around the main flux acted as short-circuited turns and absorbed the excitation, only a small portion acting to create the main flux.

Since there would be little or no main field, the m.m.f. due to armature turns would create a considerable field where the brushes are located. If the speed were high enough and the armature amperes great enough, there would be produced a considerable e.m.f. in the local circuit under the brush, causing local current to flow. Commutation of local and line current would tend to cause flashing from bar to bar and a flash-over of the motor before the main flux rises to any appreciable value to aid in commutation.

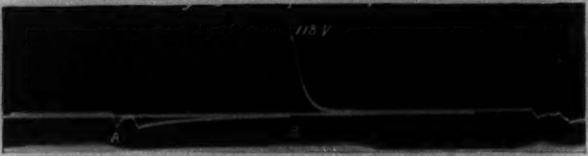
Another way of looking at it is to suppose a motor operated at a constant pressure of say 500 volts, to have its field reduced to one third or one fourth of its value, the armature being speeded up three or four times its full field-speed, and a load of two to three times normal current put on the armature. Any one familiar with the operation of 500-volt machines will appreciate at once the likelihood of a flash-over.

In order to segregate these causes and find the value of each, the following tests were made:

The railway motor was driven at various speeds, the current through the field was interrupted and made with various maximum currents and pressures in the circuit. The armature was open-circuited, the brushes, however, being well fitted to the commutator. Should causes outlined above in 1 be ruling, flashing would be produced.

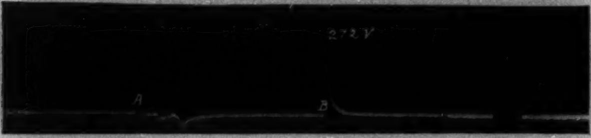
The speed and excitation under which these tests were conducted generated approximately 1500 volts across the commutator, and throughout the entire tests there was not even the slightest sign of flashing, showing that the condition outlined in 1 was not in any way ruling, and that conditions outlined in 2 were by far the ruling causes of flash-overs.

The oscillograph was employed to record the current, pressure across the armature, and the pressure across the field. The tests were taken with standard field and with special fields, and are compared on the following oscillograms: An explanation of these will suffice to bring out the facts.



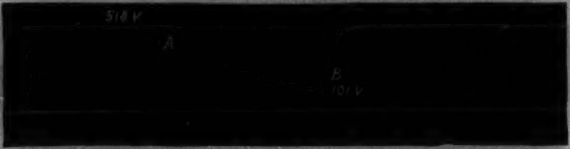
Oscillogram 8.
Standard field. Pressure across motor field. 67 amperes, 30 interruptions per min.

Oscillogram 8 shows pressure across the standard field when current is broken and circuit made again. At *A* current is interrupted and there is self-induction pressure which lingers for an appreciable time. At *B* the circuit is made again, the time from *A* to *B* is approximately one second, the motor running free on the stand at approximately one-third load and at double full-load armature speed.



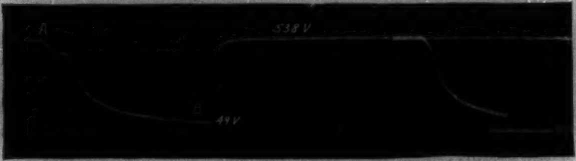
Oscillogram 9.
Special field. Pressure across field of motor. 538 volts, 60 amperes, 30 interruptions per min.

Oscillogram 9 is the same as Oscillogram 8 except the field coils are special (all short-circuiting effects removed), the effects of the solid steel frame, however, remaining. The particular point is that the self-induction pressure across the field coils is more than twice as much with special fields as with standard fields, and it will be seen later the maximum current with special fields is about half that of standard fields.



Oscillogram 10.
Standard field. 61 amperes, 30 interruptions per min.

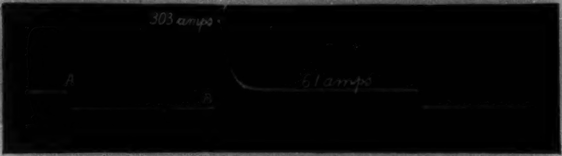
Oscillogram 10 shows the pressure across the armature with standard fields. From *A* to *B* is about one second. The pressure gradually fell and at the end of one second there was still sufficient flux to give 101 volts, or about one fifth of the field. It will be noted that at *B* a sharp peak occurs which evidently is the self-induction pressure of the armature. As soon as the current has reached its maximum, this pressure is zero. In the meantime, however, the main flux is rising slowly so that the e.m.f. wave drops down and continues to record the counter e.m.f. due to the main flux of motor. The action will be clearly seen by a slight analysis of the curve.



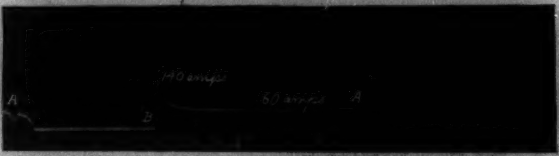
Oscillogram 11.
Special field. 60 amperes, 30 interruptions per min.

Oscillogram 11 is the same as Oscillogram 10 except the fields are special. Note how rapidly the main flux decreases, the pressure falls from 538 to 49 volts; to less than 1/10. Note also that the armature self-induction pressure record disappears and the counter e.m.f. due to main flux rises very rapidly.

Oscillogram 12 shows the current in the motor with *standard fields*. At *A* the current is interrupted; after a lapse of one second the circuit was again made at B. The motor was running free at 61 amperes and when circuit was made again the current rose to a maximum of 303 amperes, a ratio of about five times the minimum and about 50% more than the hourly rating of the motor.



Oscillogram 12.
Standard field. 518 volts, 30 interruptions per min.



Oscillogram 13.
Special field. 538 volts, 30 interruptions per min.

Oscillogram 13 shows same as Oscillogram 12 except that the fields are special. The current was interrupted at A and after a lapse of one second the circuit was made at B. The noticeable feature is that the maximum current is less than one-half that in Oscillogram 12 and only about 70% of the rated current of the motor.

Compare Oscillograms 1 and 7. In 1 the motor is operating on a steady full load. The pressure of commutation indicated is in the neighborhood of 3.5. In Oscillogram 7 there is no field excitation and simply the full-load current is passed through the armature, the armature being driven at approximately full-load speed and the pressure from brush to commutator shows 10 to 12 volts. Should the speed have been 2.5 times a full-load speed, as occurs often in practice with the minimum free-running current and maximum current passing through the armature 50% more to twice as great as the full-load current, it is reasonable to estimate that the pressure between brush and commutator bar of 45 to 60 volts may exist, or 12 to 15 times as great as when that motor is carrying a steady full-load current at full field and full pressure.

A paper to be presented at the 187th Meeting of the American Institute of Electrical Engineers, May 17-18, 1904.

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ON THE CALCULATION OF LINE BATTERIES.

BY W. E. WINSHIP.

Storage-batteries have found a legitimate use in electric railway work for the purpose of maintaining the line pressure at points distant from generating stations. There is at present very little literature on this subject, and it is the purpose of the present article to discuss the calculation of such battery installations, and to compare in a general way the results with those which may be obtained by the use of copper. Independently of maintaining the pressure on the line, obviously something toward load regulation is obtained through the use of batteries.

Line batteries are installed in either of two ways; they may be fed over a boosted feeder with the average current which would be demanded from a sub-station maintained at the same pressure as the average pressure of the battery, the latter charging or discharging the difference between the instantaneous current and that supplied over the boosted feeder, or they may be simply floated on the line; that is, connected in parallel between the feeder and rail, in which case the battery will charge when the demand on the line is light and discharge when it is heavy. Such floating batteries are a close analogy to an air chamber, or a stand-pipe on a pipe line. Obviously, if the demand varies but very slightly, there is no occasion for such a reservoir; just as obviously if the demand is variable it would be a decided advantage.

The calculation of batteries to be fed over a boosted feeder from the power-house is extremely simple when the section operated in the battery is insulated from the other feeders from the power-house. We have simply to install the number of cells corresponding to the average pressure desired at the battery location, and, when we determine the average demand at this point, decide on the size of feeder advisable and hence determine the amount of " boost." The size of the feeder should involve, however, the consideration of losses in transmission, and a balancing of this loss against the amount necessary to expend for copper.

With a floating battery the determination is not quite so obvious, and the writer purposes to take up this calculation more in detail.

We consider, first, the case of a concentrated load at a point between two stations at different pressures. Let V_1 be the pressure of Station 1; V_2 that of Station 2; I the load; r the resistance between Station 1 and the load; s the resistance between Station 2 and the load; R, the total resistance; I_1 and I_2 the currents furnished by Stations 1 and 2 respectively. Then by a consideration of the drops from 1 and 2 we have

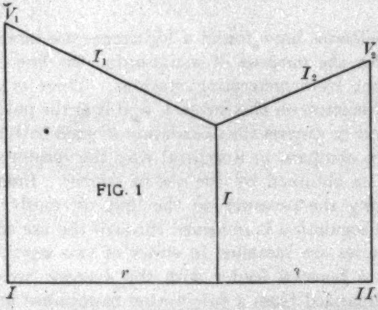

FIG. 1

$$r I_1 - s I_2 = V_1 - V_2; \text{ also,}$$

$$I_1 + I_2 = I; \text{ hence}$$

$$I_1 = \frac{[s\ I + (V_1 - V_2)]}{R} \tag{1}$$

$$I_2 = \frac{[r\ I - (V_1 - V_2)]}{R} \tag{2}$$

If we have distributed loads of i_1, $i_2 - i_n$ amperes and the resistances from Stations 1 and 2 and the points of location of these loads are $r_1, r_2, - r_n, s_1, s_2. s - s_n$ respectively where $r_m + s_m$

$= R$, the resistance between Stations 1 and 2, a generalization of equations (1) and (2) will be

$$I_1 = \frac{[\Sigma\, s\, i + (V_1 - V_2)]}{R} \tag{3}$$

$$I_2 = \frac{[\Sigma\, r\, i - (V_1 - V_2)]}{R} \tag{4}$$

Obviously,

$$I_1 + I_2 = \frac{(\Sigma\, r\, i + \Sigma\, r\, i)}{R} = \Sigma\, i = I. \tag{5}$$

where I is the sum of all the currents. It is possible to interpret these equations very simply in a geometrical diagram.

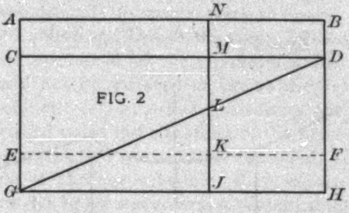

FIG. 2

In the figure $C\,G$ represents I, the total current; $G\,H$ is the resistance R; $A\,C = M\,N = K\,J = \frac{(V_1 - V_2)}{R}$ = current which would flow from Station 1 to Station 2 with no load on the line. $G\,D$ is the diagonal of the figure $C\,D\,H\,G$, and $G\,J = \Sigma\, r\, i / \Sigma\, i$. The point J thus corresponds to the center of gravity of the loads. We have then $L\,N = I_1$ and $K\,L = I_2$. The application of this diagram we shall take up below.

Preliminary to the determination of the proper location of a floating battery we shall next consider the problem: with a definite current, I, on the line at various points what is the value and location of minimum pressure? We preserve the preceding notation.

We have as the drop in pressure from Station 1

$$D = r_1 I_1$$

or from equation (1) inserting the value of I_1

$$D = -\frac{r_1^2 I}{R} + r_1 \frac{[I + (V_1 - V_2)]}{R}$$

and therefore, as the pressure on the line at any point,

$$V = V_1 - D = \frac{r_1^2 I}{R} - r\,\frac{[I + (V_1 - V_1)]}{R} + V_1$$

This equation has the form

$$y = A\,x^2 + B\,x + C$$

and represents a parabola whose axis is parallel to $O\,V$. If we make the substitution $x = x + e$ we may make this equation take the form

$$y = A\,x^2 + E$$

(6) if $$e = \frac{I\,R + (V_1 - V_2)]}{2\,I}$$

and

(7) $$E = V_1 - \frac{[I\,R + (V_1 - V_2)]^2}{4\,I\,R}$$

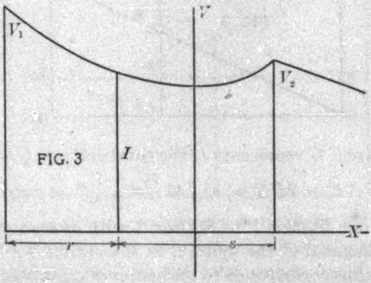

FIG. 3

E is then the minimum pressure on the line with the current I and e is its distance from Station 1 in ohms. Beyond Station 2 the drop will be simply the resistance multiplied by the current.

In general, with definite feeder and return circuit the best results as far as line pressure is concerned will be obtained by making the minimum pressure with maximum demand between Stations 1 and 2 equal to the minimum beyond 2. With a number of cars on the line it is not of course sufficient to consider the total current as concentrated at the center of gravity of all the currents and then calculate the drop to this center of gravity; determined in this manner it would be much too large. We must consider the separate currents and then determine the minimum pressure. The best point of location of the battery is, in any case, readily determined. It will plainly depend on grades and accelerations, but it may be said that the point is ordinarily between 0.66 and 0.75 of the distance from the power-house to the endof the line.

The open-circuit pressure of a battery varies from 2.07 to 2.12 volts per cell according to the acid density, temperature, and state of charge. It is usual for regulation work to allow a battery to float in approximately a three-quarter charged condition as the efficiency is best in the neighborhood of this point and the pressure fluctuations on charge and discharge are less. We shall take 2.1 volts per cell as the proper value on open circuit—it is actually a trifle less—and assume the battery as equivalent to a station operating at a pressure of 2.1 times the number of cells. The pressure will ordinarily vary in charge and discharge from 3% to 6% above and below this value, but since the battery will charge and discharge by correspondingly less amounts, our calculations will not be vitiated as far as the determination of the number of cells is concerned by taking the above value. We shall afterward point out the effect of the pressure variations.

Floating batteries are quite often installed, so that over a complete cycle of the car schedule they experience neither a net charge nor discharge excepting a certain amount necessary to make up the internal loss of the battery; this is, however, an extremely small amount, as the ampere-hour efficiency of a storage-battery is extremely good. In order for a battery to operate in this way all of the conditions of operation, including schedule, station pressure, feeder, etc., are fixed, and if any of these conditions are departed from the battery will either charge or discharge. It is obviously possible with a regular daily schedule, which may however vary from time to time during the day, so to install a battery that the net charge during the lighter hours will equal or slightly exceed the net discharge. This may be done by keeping the power-house at approximately a constant average pressure with a varying schedule, provided the battery has the proper number of cells; if, however, the net discharge over any maximum period exceeds the ampere-hour capacity of the battery, it will be necessary either to vary the pressure at the power-house or vary the number of cells of the battery with varying schedule conditions.

In this connection it is proper to note that the capacity of the battery should be reckoned approximately at the rate corresponding to the duration of all the net discharges, and not at the rates at which the separate discharges occur. This is due to the recovery element of a battery; that is, if a battery is discharged at a high rate for a short time and then allowed to recover, and the same cycle is repeated a number of times, then the ampere-hour capacity of the battery on this work very nearly approximates the capacity at the rate corresponding to the total time elapsing.

Quite usually the conditions of operation are that in early morning and late at night the schedule is less frequent than during the major part of the day; during this latter time the loads vary at different times. Generally the calculation of a battery for this sort of schedule would be based on the determination of an average running current per car under different conditions. These values would be most suitably determined from station wattmeter readings, dividing by the pressure of the station, time elapsing between readings, and the number of cars. The battery should then be so calculated that it receives a net charge during the early morning and late at night, floating during the greater part of the day and discharging during the maximum hours. The net charge should slightly exceed the net discharge to make up for the inefficiency of the battery.

With reference to the formulas (3) and (4) and their geometrical interpretation in the diagram, Fig. 2, it is obvious that if we base our calculations on the number of cars distributed on the line, each taking the average current per car, then a battery installed at the point II, of average voltage V_2, so determined that $(V_1 - V_2)/R = JK = JL$ (that is, so that K coincides with L), will neither experience a net average charge nor discharge over an interval corresponding to a complete cycle in car schedule. If during any instant the current demand of the cars departs from the average values then the battery will charge or discharge according to the variation of these currents. It will be noticed that the center of gravity oscillates in general between certain limits even if we assume that the cars all take a constant average current. In any particular case it will appear, whether it suffices to take the point J as the average of the centers of gravity, or, if it will be necessary to take several different positions, according to the number of cars on the section I to II. In case the number of cars on the section varies from time to time it is convenient to calculate separately the components of the battery discharge and obtain the mean. In the calculations all cars beyond the battery are to be considered as concentrated at Station 2.

If a battery is to float, absolutely, that is, stay at the same state of charge, an extremely simple pressure diagram may be drawn:

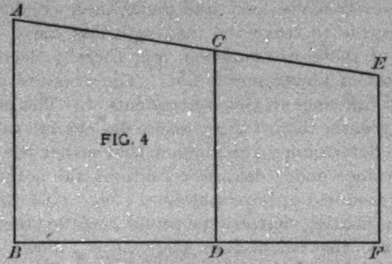

FIG. 4

Make $AB = V_1, BD = \frac{\Sigma r i}{\Sigma i}$ and $EF = RI$; then $CD = V_2$.

The problem of a floating battery may take either of the three forms: (*a*) with a given feeder and rail circuit and definite schedule determine the pressure at which a battery will float; (*b*) assuming a definite pressure at the battery location, with a definite schedule, how much copper is necessary? or (*c*) with definite copper and return circuit and a definite schedule what will be the battery charge or discharge? We are in a position to solve each of these problems, or a combination of them.

In (*a*) we determine the center of gravity of the average running currents; then in the diagram we obtain $JK = JL$. From the relation $JK = JL = (V_1 - V_2)/R$ we obtain in V_2. In (*b*), similarly, we obtain JL and from the same relation determine R. In (*c*) we obtain $JK = (V_1 - V_2)/R$ and obtain from the diagram $KL = JL - JK$, the battery discharge. We have the possibility of regulating arbitrarily either singly, or in combination, V_1 or V_2, perhaps even R, to accommodate for any schedule variation which we do not wish to impose on the battery capacity. To regulate V_2 we either cut in or out a certain number of cells or arrange certain cells for grouping, either in series or parallel.

The diagram, Fig. 2 shows also the extent to which the total instantaneous current demand comes on the power-house if we draw the diagram for the instantaneous currents of the separate cars. Very clearly, the nearer a heavy load is to the power-house the larger is the portion of this load furnished by it. If the heavy currents are demanded near to or beyond the battery, the latter furnishes the major portion of the load. If grades and starts occur near the end of a line the battery will very materially steady the power-house load; and under these conditions it will often be possible to carry the load with less units, raising the load-factor of those in operation, and thereby decrease materially the cost of power production. The pressure variation is of particular influence in this determination. The power-house load will be greater than if the pressure did not fall as it actually does, and in determining the amount that either the battery or power-house does under definite conditions the actual value of V_2 must be assumed approximately.

In case a floating battery is to be installed between two power-stations, the calculation would be as follows:—calculate the currents $J\,L$ for the battery and each power-house separately; if I is the sum of these two currents, and the battery is to float, then we must make

$$\frac{(V_1 - V_B)}{R_1} + \frac{(V_2 - V_B)}{R_2} = I \tag{8}$$

where V_B is the battery pressure.

The condition that in addition to floating on a feeder system, the battery receives current over a boosted feeder, with the same track return, is also very simply calculated.

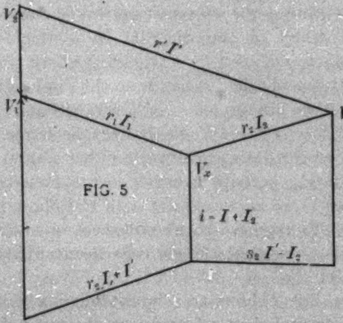

FIG. 5

Preserving the previous notation with the exception that we indicate the feeder and ground return resistances separately by subscripts $_1$ and $_2$ respectively, and indicating the booster pressure by V_x, the resistance of the separate feeder by r' and its current by I^1, we shall have in the network the currents marked in the diagram, where also, (by a consideration of pressure drop),

$$V_x = V_1 - (r_1+r_2)\, I_1 - r_2\, I^1$$
$$= V_2 - (s_1+s_2)\, I_2 + s_2\, I^1$$

Or writing $r_1+r_2 = r$; $s_1+s_2 = s$; and $r_2+s_2 = R_g$, the resistance of the return, we obtain

$$(9) \quad r\, I_1 - s\, I_2 = (V_1 - V_2) - R_g\, I^1$$

together with (10) $I_1 + I_2 = i$.

The solution of these equations is

$$(11) \qquad I_1 = \frac{\{s\, i + [(V_1 - V_2) - R_g\, I^1]\}}{R}$$

$$(12) \qquad I_2 = \frac{\{r\, i - [(V_1 - V_2) - R_g\, I^1]\}}{R}$$

Where there are several loads, i_1, i_2, $- \, i_n$ the generalization of this solution is

$$(13) \qquad I_1 = \frac{\{\Sigma\, s_1 i_1 + [(V_1 - V_2) - R_g\, I^1]\}}{R}$$

$$(14) \qquad I_2 = \frac{\{\Sigma\, r_1 i_1 - [(V_1 - V_2) - R_g\, I^1]\}}{R}$$

The condition that the battery should float is $I_2 = I^1$. Inserting this in (14) we obtain

$$(15) \qquad I_2 = \frac{[\Sigma\, r_1 i_1 - (V_1 - V_2)]}{(R - R_g)}$$

The modification in the diagram, Fig. 2 is to make the line $A\,B = R - R_g$. It should be noticed that R is the resistance of the circuit exclusive of the boosted feeder.

The solution of our problem is therefore obtained by finding the value of I_2 and supplying this current over the auxiliary feeder. The amount of " boost " is this current, I_2, multiplied by the resistance of the feeder; the return is already included in the calculations.

We have thus developed a method of calculating line batteries under various conditions. To compare absolutely the results obtainable with copper and with batteries is impossible without taking up each problem and investigating it for itself, as there are too many factors entering into the comparison. We should consider first the costs of copper alone, and copper with batteries for equivalent line voltages; also the interest and depreciation in the two cases, together with the transmission losses. The battery should be credited with the influence which the load equalization effected would have on the cost of power production; and, also, the emergency value of the battery, in case of temporary breakdown, should receive consideration. This latter value is extremely difficult to express in a money equivalent. In some cases, however, it is so important as far to outweigh any question of economy of operation. It may be stated that with fairly long lines, say lines whose ends are six to ten or twelve miles distant from a power-house, and if the cars run are relatively few in number and heavy, a battery installation will generally cost less than equivalent copper. If grades and starts occur, principally distant from the power-house, the value of the battery as a load equalizer would nearly always justify its installation.

In any event, a careful comparison of the results which can be obtained with batteries should be made with the other feasible ways of obtaining the desired ends. In case of comparison with sub-station installations we must not forget that the battery requires almost no attendance in comparison with an alternating-current sub-station.

A paper to be presented at the 187th Meeting of the American Institute of Electrical Engineers, New York, May 17–18, 1904.

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WAVE FORM VARIATIONS OF A LONG-DISTANCE LINE.

BY GEORGE H. ROWE.

The map, Fig. 5, shows the lines of the Standard Electric Company's system on which the following oscillographic curves were taken. At the head of the system, Electra, the power-house contains five 2000-kw. three-phase generators of the S.K.C. inductor type, each giving 2200 volts at 60 cycles per sec. These generators are supposed to have an inherent regulation of 3% on full non-inductive load, and 15.5% on full kilovolt-ampere load, 70% power-factor. The normal exciting current for full load, 100% power-factor, is 150 amperes at 2200 volts, with a limiting field-current of 250 amperes.

The Bay Circuit shown on Fig. 5 is fed from two banks of three transformers each, both high- and low-tension circuits of which are △-connected. The ratio of transformation is 16.4, and therefore the line pressure is above or below 36 000, depending on the load and power-factor of the system. Besides the Bay Circuit, several mining circuits extend to neighboring mining camps in Ammador and Calaveras counties. The maximum length of these is about 30 miles. They are operated independently of the Bay Circuit at about 17 000 volts. The Bay Circuit lines consisting of aluminum cables extend to San José, a distance of 113.6 miles: to Oakland by way of Mission San José, a distance of 128.8 miles, and to San Francisco by way of Alviso a distance of 153 miles. The calculated charging current of the system at 40 000 volts is about 30.3 amperes on the assumption of a sine wave. From the generators this would mean 500 amperes line current.

In Fig. 6 will be found curves of observations taken on January 16, 1904. The power curve is derived from 15-minute readings of the integrating wattmeters and is probably considerably in error, both on this account and on account of the large leading current.

It is difficult to give an accurate idea concerning the nature of the load. At Stockton are two 150-kw. synchronous motor-generator sets. The San José division contains one 250-kw. and one 150-kw. synchronous motor-generator set, both used for street railway and general power work, the generators being operated in parallel with a storage-battery. The direct-current motors range in size from 0.5 to 20 horse power and aggregate about 500 horse power. There are also induction motors operating from the lighting circuits, ranging in size from one to

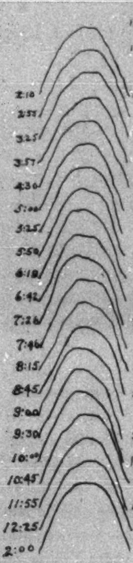

Fig 1
Electra

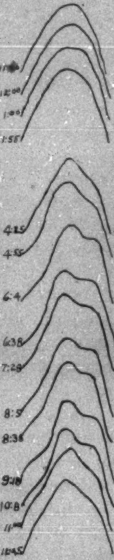

Fig 2
San José

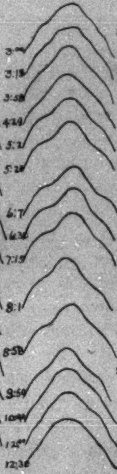

Fig. 3
Stanford

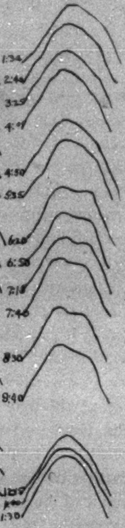

Fig. 4.
San Francisco

75 horse power, and aggregating about 300 horse power. The total peak supplied to the San José sub-station is about 1000 kilowatts.

The San Mateo division is operated from the sub-station at Redwood. Distribution is made at 6000 volts, two-phase, to neighboring small towns. Induction motors in this division range in size from one to thirty horse power, and aggregate about 500 horse power, constituting chiefly a day load. The maximum load supplied to the station occurs between the hours of five and nine p. m., giving a peak of about 400 kilowatts.

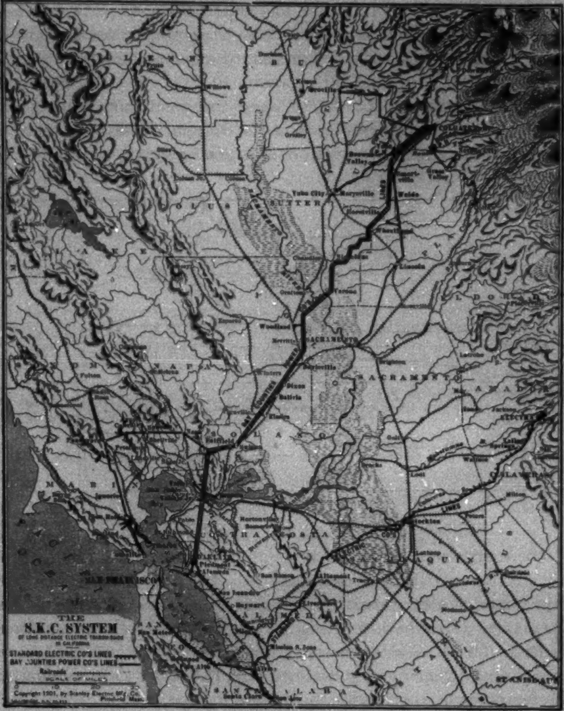

Fig. 5.

At the San Francisco sub-station, the low-tension circuits are Y-connected with the neutral grounded, and supply power at 4000 volts between lines to station *B*, and to station *C* of the San Francisco Gas and Electric Company. At station *C* are two 500-kw. synchronous motor-generator sets, and at station *B* there is one 400-kw. synchronous motor-generator set. Station *B* also contains two 1200-kw. and one 600-kw. G. E. revolving-field generators, one or more of which were operated in parallel during these tests with the Standard system, being thrown in at about 5 30 p. m. and off at about 10 30 p. m. The peak was about 2000 kilowatts. These were the conditions during the tests. Since the consolidation of the Standard and Bay Counties systems, the above description does not hold.

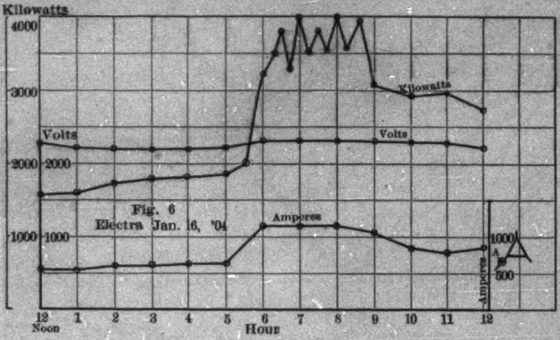

Fig. 6
Electra Jan. 16, '04

The instrument with which the following tests were made was the well known Blondel oscillograph. Briefly, it consists of two thin strips of soft iron stretched vertically between the poles of a powerful permanent magnet. Very small mirrors are attached to these strips, and each strip and mirror is placed in a tube containing glycerine or vaseline oil. The natural frequency of the instrument as given by the maker is about 1000 cycles per sec. About each of the mirrors as centers, two bobbins or coils serve to produce fields, one proportional to the current, and the other proportional to the electromotive force. The current coil required about 10 amperes to produce a convenient deflection on the plate, and the electromotive force about 0.333 ampere. A diagram through a vertical section of the instrument is shown in Fig. 7. Light from the arc *A* passes through the slits *S* through the small lens *L* in the oil tube to the mirror *M*, then through the cylindrical lens *C* to the vibrating mirror *V* and thence to the ground-glass screen or photographic plate *D G*. The mirror *V* vibrates so as to produce motion of translation of the point of light from *D* to *G*, the vibration being produced by the cam *K* operated by means of a small synchronous motor *P*. The apparatus is equipped with an electrical shutter which cuts off the return curve from *G* to *D* and further allows the taking of photographs.

The apparatus was connected in the following tests as in Fig. 8. A non-inductive resistance *R* consisting of incandescent lamps was placed in series with the pressure coil, and the current coil was shunted across the non-inductive resistance *R′* through the series resistance *R″*.

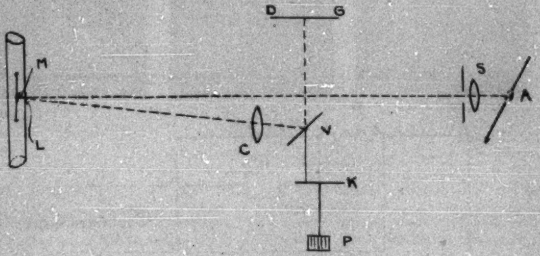

FIG. 7.

This apparatus was tried on generators of known wave form, on non-inductive loads, on leading and lagging currents supplied to a synchronous converter, and found to give consistent results.

During November 1903, a large number of observations was made at Stanford University of the electromotive force waves from the lines of the Standard company, which supplies the university with light and power. It soon became apparent that the wave form varied considerably, the distortion varying apparently with the time of day, and hence probably with the load.

On Saturday November 28th, negatives were made at approximately 30-minute intervals, beginning at 3 00 p. m. and ending at 12 30 a. m. on Nov. 29th. Tracings of these waves are shown in Fig. 3, the time of taking being given at the left of each curve. They were taken across the terminals of one of two transformers arranged to supply a 120-volt, two-phase synchronous converter, the synchronous converter being disconnected. The electromotive force varied between the limits of 88 and 103 volts during the test. Counting from the power-house, this pressure was the result of four transformations,—namely from the generator at 2200 volts to 36 000 volts at Electra; from 36 000 to 6000 volts at Redwood sub-station, and also a phase transformation from three-phase to two-phase:

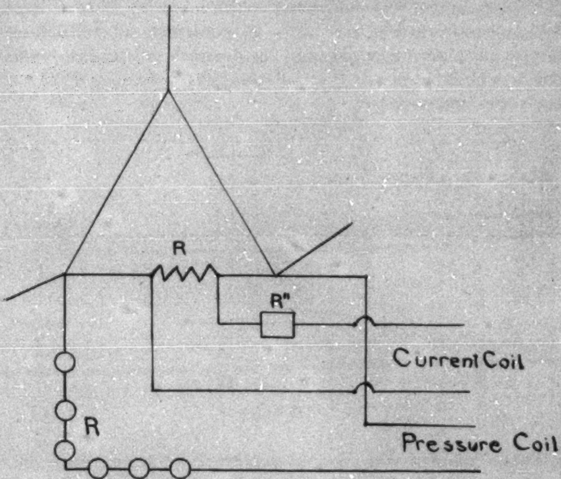

FIG. 8.

from 6000 volts to 2300 at Stanford power-house; and from 2300 volts to 90 volts in the transformer from which readings were taken. The original curve taken at 7 15 p. m. is reproduced in Fig. 9. This curve was enlarged photographically, so that the length of one cycle was 2π inches when its equation was found to be

$$y = 1.718 \sin (\theta - 0^\circ\ 20') + 0.063 \sin (3\theta + 177^\circ\ 31') + 0.138 \sin (5\theta - 29^\circ\ 15') + 0.028\ (7\theta - 90^\circ) + 0.001 \sin (9\theta - 90^\circ)$$

The positive and negative halves of the wave are not quite the same, due probably to a slight pumping of the synchronous motor operating the vibrating mirror. The above equation represents two half-waves similar to the positive half-wave. It will be noticed that the 12 30 wave approaches sine-wave form.

On Thursday December 31st 1903, the waves represented in Fig. 2 were taken at San José. The pressure coil of the instrument was placed across the 2300-volt mains through lamps in series, and the current coil was shunted across a non-inductive resistance placed inside the △. The distortion first appeared at 4 36, when the lighting load began to come on, the day being dark and rainy. This curve together with one taken at 7 28 are reproduced in Figs. 10 and 11. The wave taken at 7 28 was enlarged so that one wave length was 2π inches in length, and analyzed graphically, for the first three harmonics. The equation is as follows:

FIG. 9.

FIG. 10.

FIG. 11.

$$y = 2.89 \sin(\theta - 17') + .0283 \sin(3\theta + 56^\circ\, 58') + 0.22 \sin(5\theta + 180^\circ\, 6')$$

The small motor operating the mirror pumped badly. The above equation shows a 9.7% third harmonic and a 7.6 fifth harmonic. Since synchronous-motor load predominated, the current leads by a small angle.

On Saturday January 16th 1904, the curves shown in Fig. 1 were taken at Electra. The connections were the same as those at San José, the electromotive force being taken across the 2300-volt mains and the current being shunted across a non-inductive resistance inside the △. The pressure waves show little variation throughout the night. From 2 45 p. m. until 12 00 midnight two generators were in parallel. At 2 10 p. m. and at 2 00 a. m. are shown the wave forms of a single generator. The original curves taken at 2 10 p. m. when the load was light, and at 7 41 p. m. when the peak occurred, are shown in Figs.

12 and 13. Large leading current is shown in both. The average output at the peak is 3700-kw., the line current 1150 amperes and the pressure 2280. The power-factor by switchboard instruments would therefore be 80%, indicating a lead of 37 degrees. The curve, Fig. 13, shows greater lead than this. The calculated charging current for the Bay Circuit is about 30.3 amperes on the assumption of a sine wave with a corresponding generator current of 500 amperes. The line current at 2 10 p. m. was 550 amperes. The power-factors figured from the curves and from instrument readings do not agree. The curves show a gradual change in lead angle from 2 10 to 7 41 while the wattmeters indicate only a small change in power-factor. The wattmeters probably do not indicate correctly on the very large leading current.

In Fig. 14 are shown the waves of electromotive force and current supplying the mining circuits at 17 000 volts, the lagging induction-motor load neutralizing the charging current.

Fig. 12.

Fig. 13.

Fig. 14.

The curves reproduced in Fig. 4 were taken at the San Francisco sub-station on Feb. 6th. The transformers, as before stated, are here Y-connected to the 4000-volt mains of the San Francisco Gas and Electric Company. The pressure coil was connected between the neutral and one line, the current coil being shunted across a non-inductive resistance as in the preceding cases. The original curve reproduced in Fig. 15 was taken at 7 40 p. m. The last wave taken at 1 30 is seen to be practically the generator wave as shown at 2 10, Fig. 1. Although these observations were not taken simultaneously, load conditions were probably nearly the same for all of the four sets. The load curves also were much the same as those plotted for Jan. 16th.

Figs. 2 and 4 are seen to be similar. Had a fourth set been taken at Redwood from the three-phase circuit, it would have been probably similar to 2 and 4. Pressures corresponding to the waves as given in Figs. 1, 2, 3, and 4 are given in the following table:

Electra.	San José.	Stanford.	San Francisco.
2200	2200	98	2500
2200	2240	96	2500
2200	2240	103	2500
2200	2220	95	2500
2200	2220	92.5	2500
2200	2300	90	2520
2200	2340	88.5	2600
2300	2300	89	2600
2300	2300	89	2600
2300	2300	89	2600
2300	2320	90.5	2600
2280	2340	90	2590
2280	2320	95	2500
2280	2300	96	2500
2280	2220	94	2500
2280			
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2200			
2200			

While taking observations at San José, there was considerable pulsation on the line, especially at 6 38, due to the generators of the San Francisco Gas and Electric Co. which were operating in parallel. The effect as observed on the ground-glass screen of the instrument was a see-sawing of the two peaks of the wave: when the amplitude of one increased, that of the other diminished, and the reverse. This phenomenon suggested that perhaps the distortion was entirely due to the influence of the San Francisco generators This assumption is, however, not borne out by Fig. 16 taken at San Francisco with the generators disconnected. The wave form of the San Francisco generators is shown in Fig. 17. A moving film attachment to the instrument would give much valuable information on surging caused by switching, short circuits, or pulsations due to any cause.

The effect of the variable wave form in low-tension circuits as shown in Fig. 3 on the accuracy of an induction meter was investigated and found,to be negligible. It is, however, probable that the accuracy would be considerably affected on wave forms like those shown in Figs. 2 and 4, depending on the amplitude and phase relations of the harmonics.

The variation of the core losses has not yet been determined. The ratio of maximum to virtual in curve 7 28, San José, is 1.4, and the ratio in curve taken at 11 p. m. is 1.55, indicating a 10% variation and therefore, a probable inverse variation in core loss of like amount. Maximum pressures as derived either from spark determination or from transformer ratio may also be considerably in error.

There is reason to believe that greater variations occur on long lines which are more fully loaded. The large wattless currents sometimes observed in synchronous motors and synchronous converters with fields adjusted for minimum current are probably due to wave-form differences. These together with pulsation difficulties could be satisfactorily observed with a moving-film oscillograph.

Where so many disturbing elements are present it is difficult to select the predomniating cause of distortion. Doubtless it is due to the influence of synchronous motors of dissimilar wave form, to alternators of dissimilar wave form, and largely to the pulsation of reactance. Distortion caused by change of permeability due to hysteresis will vary with the line drop as pointed out by Mr. H. A. Pikler.* This appears to be true in this case.

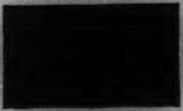

FIG. 15

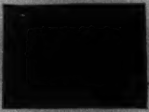

FIG. 16.

FIG. 17.

The following summary of conclusions is suggested for consideration:

1. Advisability of substituting induction motors for synchronous motors on lines with large charging current, and the use of the latter as synchronous compensators properly distributed.

2. The impossibility of designing machines to suit all wave forms.

3. The oscillograph offers the best means of studying variations in wave form, amplitude factor, surgings, pumping, etc.

4. The best form of oscillograph for this work is the moving-film or falling-plate type, without the synchronous motor attach-

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* See *Elec. World*, Aug. 8, 1903.

ment. It should be insulated for use on high pressures, and fitted with non-inductive metallic resistances.

5. The variation in amplitude factor in a long line may be considerable.

6. Core-loss variation may also be considerable.

7. The charging current may differ from that calculated on the assumption of a sine law.

8. Assumption of sine-wave form on long lines and applied to insulator and insulation testing may lead to error.

9. Variation of wave form partly due to distorting effect of hysteresis loop under different conditions of load.

The author wishes to acknowledge the courtesy of Professor J. N. LeConte in allowing the use of his harmonic analyser, and for valuable assistance in its use. The author also wishes to acknowledge the assistance of Mr. Joseph Mini, Jr., whose practical experience rendered his services particularly efficient.

A paper to be presented at the 187th Meeting of the American Institute of Electrical Engineers, New York, May 17-18, 1904.

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DATA AND TESTS ON A 10 000 CYCLE-PER-SECOND ALTERNATOR.

BY B. G. LAMME.

In the early part of 1902, M. Leblanc, the eminent French engineer, was in this country, and spent considerable time at the Westinghouse Electric & Manufacturing Company's works at East Pittsburg. M. Leblanc was very much interested in certain special telephone work, and in connection with such work he desired for experimentation a current of very high frequency. He took up with the writer the question of building a successful alternator for generating current at frequencies between 5000 and 10 000 cycles per second. He was informed that the machine would necessarily be of very special construction, but that it was not an impossible machine. Later he took up the matter with Mr. Westinghouse, who, upon receiving satisfactory assurance that such a machine was possible, advised that the generator be built. A preliminary description of the general design was given M. Leblanc before he returned to Paris. He was somewhat surprised at certain of the features proposed, especially at the fact that an iron-cored armature was considered feasible for a frequency of 10 000 cycles per second.

The machine was designed and built on practically the lines of the preliminary description furnished M. Leblanc. The frequency being so abnormal, the writer believes that many features in the machine, with the results obtained, will be of scientific interest, and therefore the data of the machine, and the tests obtained are presented herewith.

The starting point in this machine was the sheet-steel to be used in the armature. No direct data were at hand showing losses in sheet-steel at such high frequencies, nor was there at hand any suitable apparatus for determining such losses. As preliminary data, tests at frequencies up to about 140 cycles per second were used, and results plotted in the form of curves; these results were plotted for different thicknesses of sheet-steel. Also, tests were obtained showing the relative losses due to eddy currents and hysteresis, and these were plotted, taking into account the thickness of the sheets. These data were not consistent throughout; but the general shape of the curves was indicated, and in this way the probable loss at the frequency of 10 000 cylces per second was estimated for the thinnest sheet-steel which could be obtained. The steel finally obtained for this machine was in the form of a ribbon about 2 in. wide, and about 0.003 in. thick, which was very much thinner than any steel used in commercial dynamos or transformers which varies from 0.125 to 0.0280 inch. Therefore the machine had to be designed with the intention of using this narrow ribbon of steel for the armature segments.

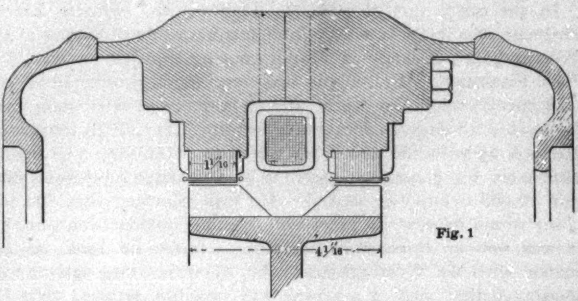


Fig. 1

A second consideration of great importance in the construction of such a machine is the number of poles permissible for good mechanical construction. For instance, at 3000 revolutions—which was adopted as normal speed—the number of poles required is 400 for 10 000 cycles per second. The frequency, expressed in terms of alternations per minute, multiplied by the pole-pitch in inches, gives the peripheral speed in inches. At 1 200 000 alternations per minute (or 10 000 cycles per second) and a pole pitch of 0.25 in., for example, the peripheral speed of the field will be 25 000 feet per minute. It was therefore evident that either a pole construction should be adopted which would stand this high peripheral speed, or the pole-pitch should be less than 0.25 in. It was finally decided that an inductor type of alternator would be the most convenient construction for this high frequency; with the inductor type alternate poles could be omitted, thus allowing 200 pole projections, instead of 400. The field winding could also be made stationary instead of rotating, which is important for such a high speed. This construction required a somewhat larger machine for a given output than if the usual rotating type of machine were adopted; but in a machine of this type where everything was special the weight of material was of comparatively little importance, and no attempts were made to cut the weight or cost of the machine down to the lowest possible limits.

The following covers a general description of the electrical and magnetic features of the machine:

Armature.—The armature was built up in two laminated rings dovetailed into a cast-iron yoke, as indicated in Fig. 1.

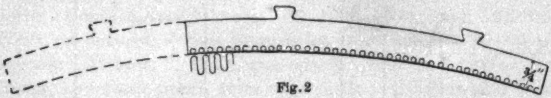


Fig. 2

The laminations were made in the form of segments dovetailed to the cast-iron yoke (Fig. 2). Special care was taken that the laminations made good contact with the cast-iron yoke, as the magnetic circuit is completed through the yoke.

The armature sheet-steel consisted of plates of 0.003 in. thickness. The sheet-steel was not annealed after being received from the manufacturer; it was so thin that to attempt annealing was considered inadvisable. To avoid eddy currents between plates each segment was coated with a thin paint of good insulating quality. This painting was a feature requiring considerable care and investigation, as it was necessary to obtain a paint or varnish which was very thin, and which would adhere properly to the unannealed laminations. These laminations had a bright polished appearance quite different from that of ordinary steel. They were so thin that the ordinary paint or varnish used on sheet-steel made a relatively thick coating, possibly almost as thick as the plates themselves. A very thin varnish was finally obtained which gave a much thinner coating than the plate itself, so that a relatively small part of the armature space was taken up by the insulation between plates.

Each armature ring or crown has 400 slots. Each slot is circular and 0.0625 inch diameter (Fig. 3). There is 0.03125 inch opening at the top of the slot into the air-gap, and the thickness of the overhanging tip at the thinnest point is 0.03125 inch.

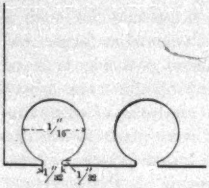


Fig. 3

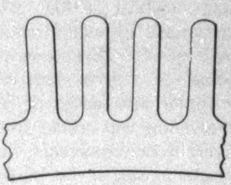


Fig. 5

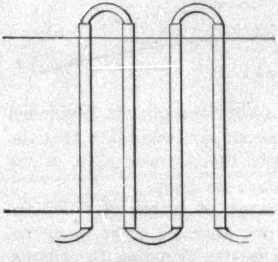


Fig. 4

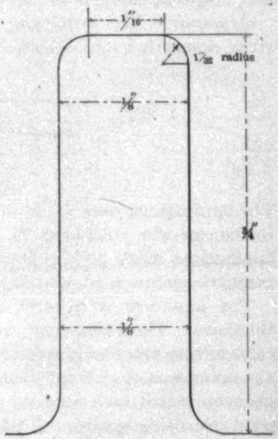


Fig. 6

The armature winding consists of No. 22 wire, B & S gauge, and there is one wire per slot. The entire winding is connected in series (Fig. 4). The measured resistance of the winding is 1.84 ohms at 25° cent.

After the sheet-steel was built up in the frame, it was ground out carefully. The laminations were then removed, all burred edges taken off and the laminations again built up in the frame. The object of this was to remove all chances of eddy currents between the plates due to any filing or grinding. The finished bore of the armature is 25.0625 inch.

Field or Inductor.—This was made of a forged-steel disc 25 in. diameter turned into the proper shape, and the poles were formed on the outside by slotting the periphery of the ring. The general construction is indicated in Figs. 1 and 5. The poles were 0.125 in. wide and about 0.75 in. long radially and were round at the pole-face. Fig. 6 shows the general dimensions of a pole.

The field winding consisted of No. 21 wire, B. & S. gauge. There were 600 turns total arranged in 30 layers of 20 turns per layer. The field coil after being wound was attached to a light brass supporting ring. The general arrangement of the field or inductor, armature yoke, and bearings, is as indicated in Fig. 1. The measured resistance of the field winding is 53.8 ohms at 25° cent.

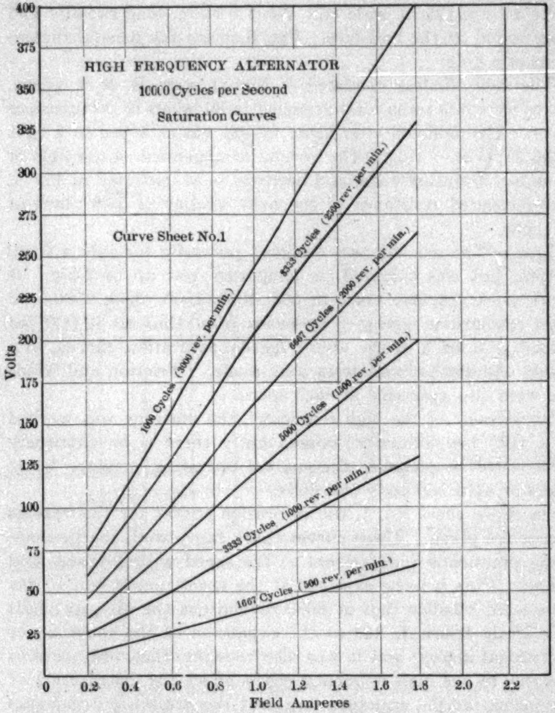
Tests.—The machine was designed primarily for only a small output, but was operated on temporary test up to 2-kw. A series of curves were taken at 500, 1000, 1500, 2000, 2500, and 3000 revolutions, giving frequencies from 1667 to 10 000 per second. At each of the above speeds, saturation curves, iron losses, and short-circuit tests were made. Friction and windage were also measured at each speed.

On account of the high frequency, the machine was worked at a very low induction; consequently there is an extremely wide range in pressure, the normal operating pressure being taken at approximately 150 volts.

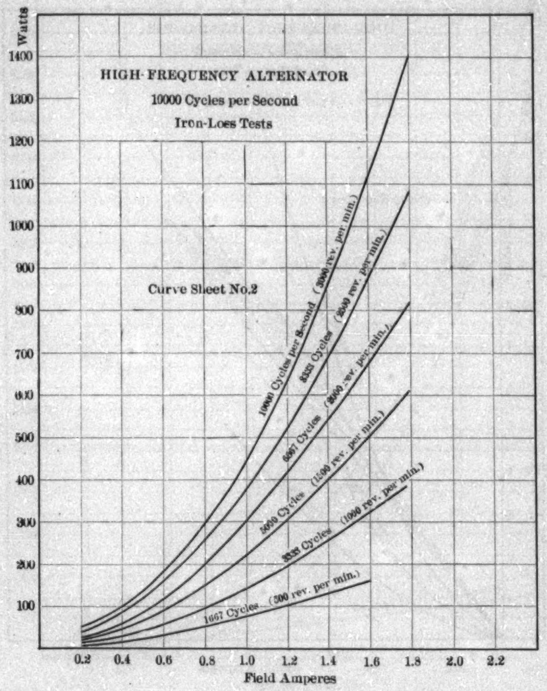
On curve sheet No. 1, the saturation curves for the various speeds are given. These curves check fairly well, the pressure being practically proportional to the speed with a given field charge. This is to be expected at the lower speeds, but it was considered possible that at 3000 revolutions the air-gap might be slightly lessened, due to the expansion of the rotor under centrifugal action; and it was also thought that eddy-current loss due to the high frequency might affect the distribution of magnetism at the armature face; but the armature iron losses were comparatively small, and there appeared to be no such effect. Also there appears to be no effect due to expansion at high speed. The air-gap specified for this machine is 0.03125 in. on each side or 0.0625 in. total gap. A very small variation in the diameter of the inductor or the bore of the armature would make a relatively large per cent. in the effective air-gap.

Therefore no reliable calculations can be made on the saturation curves of this machine based upon the specified air-gap.

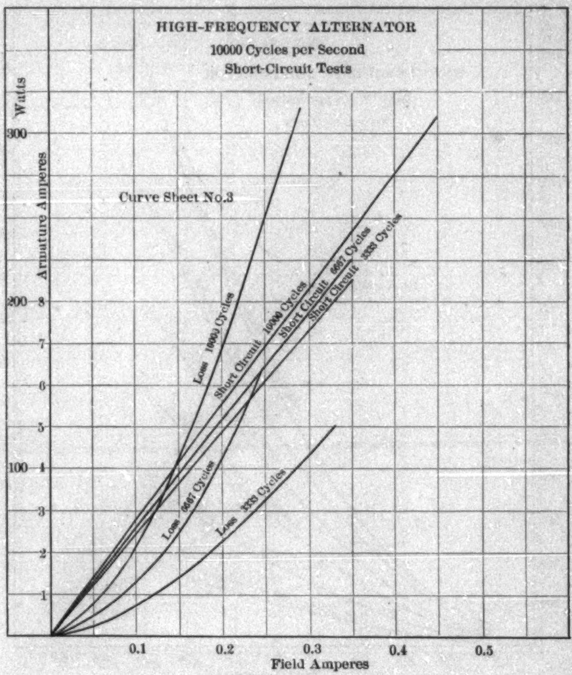
Curve sheet No. 2 shows the iron losses at various speeds from 500 to 3000 rev. per min.—1667 to 10 000 cycles per second. These losses are plotted in terms of watts for a given exciting current. These curves show a rather unexpected condition as regards the losses. According to the original data showing the relative losses due to eddy currents and hysteresis, the eddy-current loss even with these thin plates should have been much higher than the hysteresis loss, but these iron-loss curves show



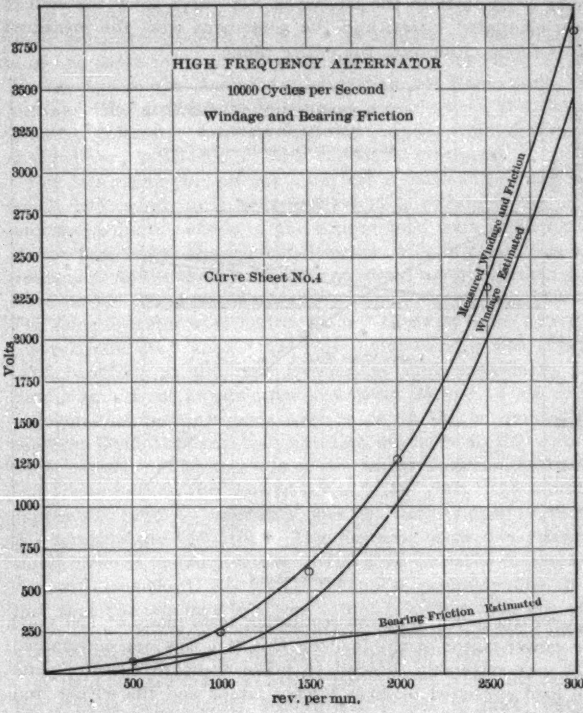
losses with a given field charge almost proportional to the frequency, which is the ratio that the hysteresis loss alone should show. As the eddy-current loss varies as the square of the frequency, the writer expected this to be a large element in the total iron loss, especially at the higher inductions. The six curves shown on this test-sheet are fairly consistent with each other, but it should be remembered that in making measurements of such abnormal apparatus little discrepancies in the curves could easily creep in. For instance, in the saturation curve a series of experiments were first made to find whether usual types of voltmeters were satisfactory, and a num-



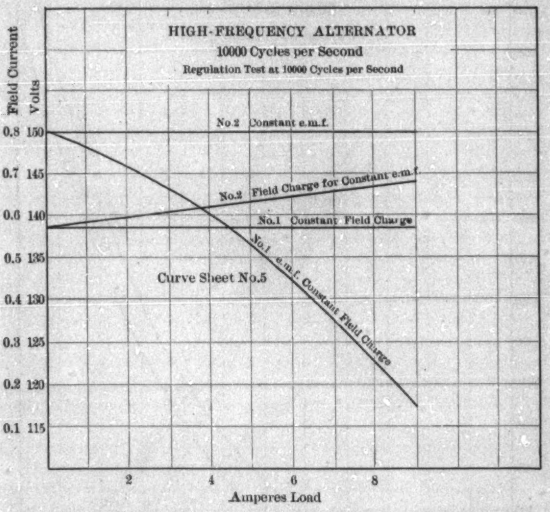
ber of different methods for checking these readings were used. In determining the iron losses in curve sheet No. 2, the machine was driven by a small motor and the losses measured with different field charges. Under most conditions of test the iron loss was a small element of the total loss, and therefore slight variations in the friction loss would apparently show large variations in the iron losses. Also the fly-wheel capacity of the rotating part of the alternator was comparatively high. Therefore, if there are any variations in the circuits supplying the driving motor, there would tend to be considerable fluctuations in the power supplied. Considering all the conditions of test, the curves appear to be remarkably consistent.



Curve sheet No. 3 shows the short-circuit curves at speeds of 1000, 2000, and 3000 rev. per min., or frequencies of 3333, 6667, and 10 000 cycles per second, respectively. It should be noted that at a given frequency the short-circuit current is proportional to the field current over the entire range measured, but that the short-circuit current is not the same for the same field current at the various frequencies. According to these curves the current on short circuit increases somewhat with the given field charge as the frequency is increased.



Curve sheet No. 4 shows the measured windage and friction losses plotted at speeds from 500 to 3000 rev. per min. This curve indicates clearly that the windage is the principal friction loss at the higher speeds. The writer has added two curves, one showing the estimated bearing friction loss, and the other the estimated windage, based upon the assumption that the bearing friction varies directly as the revolutions and the windage loss with the third power of the revolutions. The small circles lying close to the measured loss curve show the sum of these estimated losses, and the agreement with the measured loss is fairly close over the entire range.



Curve sheet No. 5 shows regulation tests made at 150 volts. The power-factor of the load on this test was not determined, as it was extremely difficult to make accurate measurements. The load consisted of incandescent lamps and the wiring from the machine to the lamps was non-inductive for the usual frequencies; but at the abnormal frequency of 10 000 cycles per second it is more difficult to obtain a true non-inductive load with ordinary apparatus. The tested regulation indicates that the load was practically non-inductive.

In first undertaking tests on this machine there was considerable difficulty in measuring the pressures. It was found that at a frequency of 10 000 cycles per second the Weston voltmeter did not work satisfactorily. Practically the same deflection was obtained on the high and low scales of a 60-120 volt Weston alternating-current direct-current voltmeter with the same pressure.

Very good results were obtained by the use of a form of static voltmeter devised by Mr. Miles Walker. This voltmeter is of the same form as the static wattmeter described by Mr. Walker before the AMERICAN INSTITUTE OF ELECTRICAL ENGINEERS, May, 1902.* Tests were also made with the Cardew hot-wire voltmeter with the high frequencies, and the results checked very satisfactorily with the static voltmeter.

For measuring the current a current dynamometer was used which had wood upright supports and a celluloid dial. The only metal parts outside of the copper coils were brass screws It was found that the current dynamometer is not affected by frequency, unless there are adjacent metal parts in which eddy currents can be generated which react upon the moving element. The dynamometer used had but a few turns in order to reduce the pressure drop across it. This dynamometer was checked very carefully at different frequencies and apparently gave similar results for any frequency between 25 and 10 000 cycles.

Several temperature tests were made on this machine. The heaviest load on any test was 13.3 amperes at 150 volts, or 2-kw. output. This test was of two hours' duration, and at the end the armature iron showed a rise of 16° cent.; the armature copper 21° cent. by resistance, and the field copper 17.3° cent. Air temperature 19° cent. The machine showed a relatively small increase in temperature at this load over the temperature rise with one-third this load. This was probably due to the fact that the windage loss was so much higher than the other losses of the machine that the temperature was but little affected by the small additional loss with increase in load.

Attempts were made to utilize the current from this machine for various experiments, but difficulty was at once found in transforming it. At this high frequency no suitable iron-cored transformer was available. Transformers with open magnetic circuits were tried and operated better than those with iron cores but were still rather unsatisfactory. It was decided that nothing could be done in this line without building special transformers.

[*TRANSACTIONS of the A. I. E. E., Vol. XIX, p. 1035.]

Among the few experiments made was that of forming an arc with current at this high frequency. This arc appeared to be like an ordinary arc so far as the light was concerned, but had a very high-pitched note corresponding to the high frequency. This note was very distressing to the ears.

This machine is in reality of the nature of a piece of laboratory apparatus; and at present it has no commercial value. It was designed primarily for scientific investigation, and appears to be a very good machine for that purpose.

A paper to be presented at the 187th Meeting of the American Institute of Electrical Engineers, New York, May 17-18, 1904.

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THE SINGLE-PHASE INDUCTION MOTOR.

BY WILLIAM S. FRANKLIN.

Fig. 1 represents a rotor having Z conductors, surrounded by a stator having a two-pole winding of N conductors spread over two opposite arcs each of 90° span. Each rotor conductor is supposed to be short-circuited through the conductor lying opposite to it so that the rotor presents $\frac{Z}{2}$ independent circuits each of resistance R. In case the rotor conductors are short-circuited by end rings, as in the squirrel-cage rotor, it is an easy matter, knowing the resistance of each rotor conductor and the resistance per unit length of the short-circuiting ring, to calculate the value of R for an equivalent winding of the type here specified.[1]

This rotor whether standing or running is magnetically stationary and magnetically equivalent to a rotor such as shown in Fig. 2 having a concentrated winding $A\,A'$ of $\frac{Z}{4}$ turns each of resistance R, and a concentrated winding $B\,B'$ of $\frac{Z}{4}$ turns each of resistance R*.

Consider an alternating flux, Φ, Fig. 2, stationary in space but harmonically distributed around the periphery of the rotor as it flows into the rotor from the stator, or into the stator from the rotor. Let E_0 be the pressure induced in the N conductors

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[1] In fact R is equal to the resistance of two conductors of a squirrel-cage winding plus z/π times the resistance of an arc of one of the short-circuiting rings of which the length is equal to the diameter of the ring.

*This result is reached by integrating the magnetomotive force of $Z/2$ independent phases of alternating current around the periphery of the motor.

($N/2$) turns of the stator winding by this alternating flux and E_1, the pressure which this flux induces in the $Z/2$ conductors ($Z/4$) turns of the rotor winding B. Then $E_0/E_1 = 0.901 \times N/Z/2 = 1.802 \times N/Z$, 0.901 being the ratio of the chord to the arc of 90°†. Therefore to give a 1:1 ratio of transformation between stator winding and rotor winding B; that is, to make $E_0 = E_1$, we may change the stator winding to give $N = Z \div 1.802$. Along with this change of N we may imagine

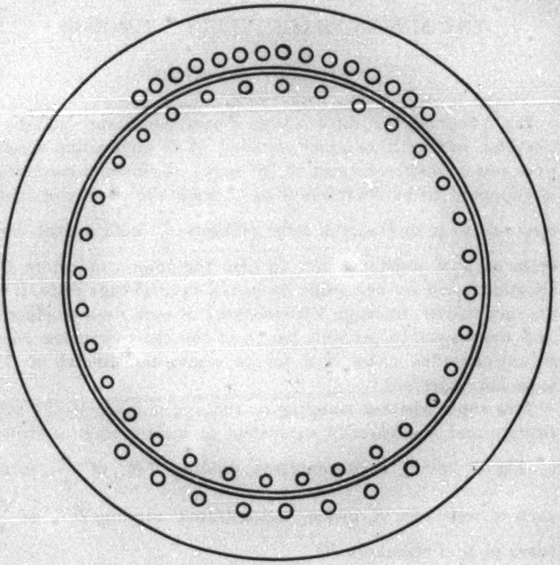

Fig 1.

the value of the supply pressure to be changed in the same ratio, and the sectional area of the wire used in the stator winding to be changed in the inverse ratio. In this way any given single-phase motor with any type of rotor and having a stator winding of any number of turns spread over any arc may be reduced to the equivalent 1:1 arrangement, and therefore the following theory is worked out for this 1:1 arrangement.

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†. See Franklin and Williamson, pages 121 to 123.

It is to be noted that the flux Φ, Fig. 2, which is stationary in space but alternating at a given frequency f induces the same pressure in any winding (distributed or concentrated) as woulp be induced in the same winding by a flux having a constant value equal to the maximum value of Φ but rotating at a speed equal to the frequency f, provided that the flux is in each case harmonically distributed around the periphery of the rotor.

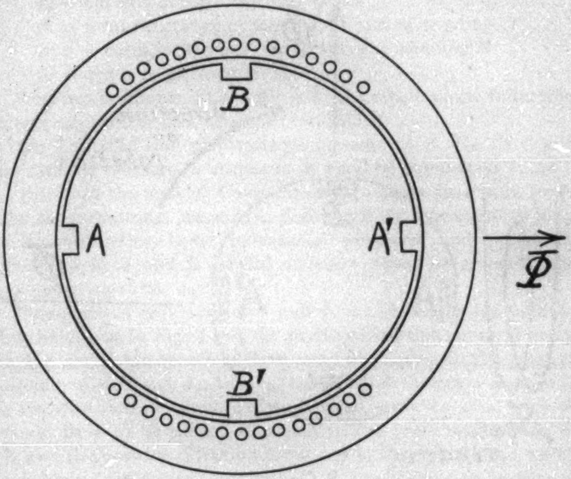

FIG. 2.

The special use to be made of this fact later is that, from it, we know that the pressure induced in the B winding by pulsations of Φ is to the pressure induced in the A winding by the turning of the rotor through Φ, as the frequency of pulsation of Φ is to the speed of rotation of the rotor.

It is furthermore to be noted that the windings A and B, Fig. 2, which represent magnetically the actual rotating windings of the rotor, are stationary, and therefore all electromotive forces induced in rotor windings and all rotor currents are to be considered to have the same frequency as the electromotive force applied to the stator windings.[1]

Fig. 3 represents diagramatically the stator winding S' of a single-phase induction motor and the two stationary windings A and B which represent the moving rotor windings at each instant. When a periodic pressure E_0 acts upon any electromagnetic system the electric and magnetic effects produced are of the same periodicity. The most completely general periodic flux through the rotor Fig. 3 consists of two periodic fluxes Φ and Φ_c perpendicular to each other in space and of which the phase difference in time may be anything whatever. We assume these periodic fluxes Φ and Φ_c to be harmonic, and we shall develop the theory of the single-phase induction motor upon these fluxes as a basis. We shall call a pressure which is induced by the time variation of a flux a *pulsational pressure*, and a pressure which is produced by the cutting of flux by motion a *rotational pressure*.

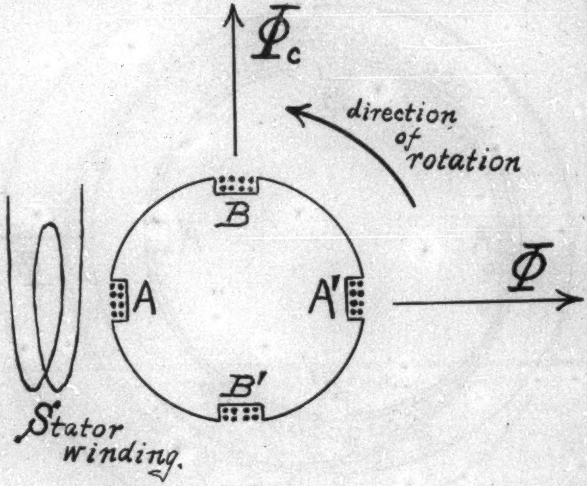

FIG. 3.

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[1] See Franklin and Williamson, pages 244 and 245.

Notation.

E' = primary impressed pressures.
E_0 = part of primary pressure which is over and above the the part used to overcome primary resistance R' and primary magnetic leakage reactance X.*
f = primary frequency
n = rotor speed
M = magnetizing current in primary
I_0 = load current in primary
I_1 = current in rotor winding A
I_2 = current in rotor winding B
R = total resistance of winding A and of winding B
$Z/2$ = turns in winding A and turns in winding B
N = turns in primary winding

The two fundamental equations of the single-phase induction motor express the following two conditions:

(a) That the sum of rotational pressure in A due to Φ and pulsational pressure in A due to Φ_c shall be equal to $R I_1$, and:

(b) That the sum of the rotational pressure in B due to Φ_c and the pulsational pressure in B due to Φ shall be equal to $R I_2$.

In considering these pulsational pressures and rotational pressures in A and B careful attention must be given to the matter of algebraic signs.

The positive directions of Φ and Φ_c are chosen as represented by the arrows in Fig. 3 and the positive directions around windings A and B are chosen as customary in relation to the positive directions of Φ and Φ_c; that is, the positive direction around A is the direction in which a right-handed screw co-axial with Φ_c would have to be turned to travel in the positive direction of Φ_c and the positive direction around B is the direction in which a right-handed screw co-axial with Φ would have to be turned to travel in the positive direction of Φ. Therefore the positive direction around A is up A and down A', and the positive direction around B is up B and down B'.

With this choice of signs the pulsational pressure in A due to Φ_c is equal to $2 \pi f \frac{Z}{2} \Phi_c / \sqrt{2}$ where Φ_c represents the maximum value of the cross-flux, and this pulsational pressure is

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*When the magnetizing current of a transformer is small compared with load current, primary and secondary currents are nearly in phase and all magnetic leakage reactance, both primary and secondary, may be located in the primary. See Franklin and Williamson, page 176.

90° behind Φ_c in time phase. Therefore this pulsational pressure in A may be written

$$-j \cdot \frac{\pi f Z \Phi_c}{\sqrt{2}}$$

Similarly the pulsational pressure in B due to Φ is

$$-j \cdot \frac{\pi f Z \Phi}{\sqrt{2}}$$

The rotational pressure in A due to Φ is in phase with Φ and positive when Φ is positive. It is therefore equal to

$$+ \frac{2 \pi n \frac{Z}{2} \Phi}{\sqrt{2}} \text{ or to } + \frac{\pi n Z \Phi}{\sqrt{2}}$$

The rotational pressure in B due to Φ_c is in phase with Φ_c and negative when Φ_c is positive. Therefore it is equal to

$$- \frac{\pi n Z \Phi_c}{\sqrt{2}}$$

Therefore we have the fundamental equations:

$$R I_1 = + \frac{\pi n Z \Phi}{\sqrt{2}} - j . \frac{\pi f Z \Phi_c}{\sqrt{2}} \qquad (1)$$

$$R I_2 = - \frac{\pi n Z \Phi_c}{\sqrt{2}} - j . \frac{\pi f Z \Phi}{\sqrt{2}} \qquad (2)$$

It is desirable to transform these two fundamental equations so as to eliminate the two quantities Φ and Φ_c. To this end we may write $k I_1$ for $\frac{\pi Z \Phi_c}{\sqrt{2}}$ where k is a constant, This is evident when we consider that the flux Φ_c is produced by the current I_1. Furthermore we may write E_0 for $j . \frac{\pi f Z \Phi}{\sqrt{2}}$ inasmuch as the pulsational pressure in B is opposite to E_0 in phase, and equal to E_0 if we make N equal to $Z/1.802$. Therefore equations (1) and (2) become

$$R\,I_1 = -j.\,\frac{n\,E_0}{f} - j\,k\,f\,I_1 \qquad (3)$$

$$R\,I_2 = -k\,n\,I_1 - E_0 \qquad (4)$$

Now I_1 is wholly a magnetizing current and it is therefore comparatively small in value so that as a first approximation $R\,I_1$ may be placed equal to zero. The effect of $R\,I_1$ upon the performance of the motor will be considered later. Therefore equation (3) becomes:

$$o = n\,E_0 + k\,f^2\,I_1 \qquad (5)$$

or

$$I_1 = -\frac{n\,E_0}{k\,f^2} \qquad (6)$$

Substituting this value of I_1 in equation (4) and solving for I_2 we have:

$$I_2 = \frac{E_0}{\left(\frac{f^2}{n^2 - f^2}\right) R} \qquad (7)$$

or, since $I_0 = -I_2$ we have

$$I_0 = \frac{E_0}{\left(\frac{f^2}{f^2 - n^2}\right) R} \qquad (8)$$

which shows that the load current, I_0, in the primary may be thought of as being produced by the useful part E_0 of the primary pressure through a fictitious resistance $\left(\frac{f^2}{f^2 - n^2}\right)R$. This fact enables us to apply a modified form of the Heyland* diagram to the calculation of the performance curves of the single-phase induction motor.

Before proceeding to this application of the Heyland diagram it is interesting to consider the relation between Φ and Φ_c on the assumption that $R\,I_1$ is equal to zero. Upon this assumption equation (1) reduces to:

$$o = n\,\Phi - j\,f\,\Phi_c \qquad (9)$$

or

$$\Phi_c = -j.\,\frac{n}{f}.\,\Phi \qquad (10)$$

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*See Franklin and Williamson, page 256.

which shows that Φ_c is 90° behind Φ in time phase and that the ratio Φ_c/Φ is equal numerically to n/f. Therefore the cross-flux Φ_c is zero at standstill and it is equal to Φ at synchronous speed. Furthermore at standstill only the simple pulsating flux Φ passes through the rotor, as the rotor speeds up the resultant flux through the rotor becomes an elliptically rotating flux, and when $n = f$ the resultant flux through the rotor becomes a rotating flux of unvarying value exactly as in the polyphase induction motor. Furthermore the magnetizing current I_1 being proportional to and in phase with Φ_c is 90°

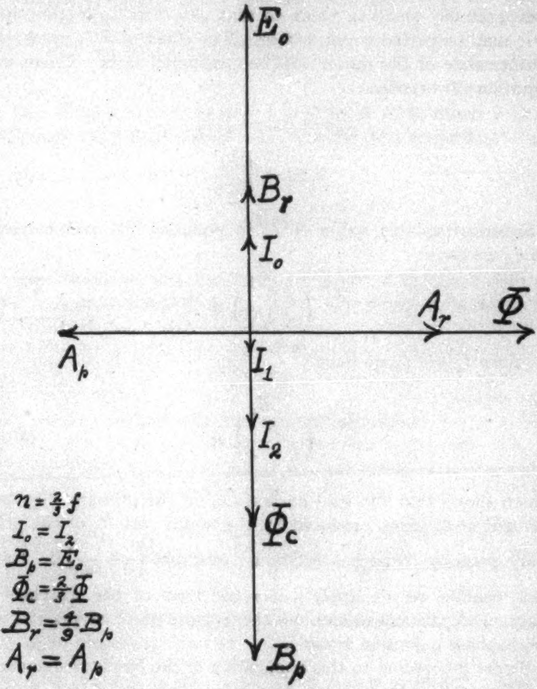

FIG. 4.

behind Φ or 180° behind E_0 in time phase; it is equal to zero at standstill, and it increases in proportion to rotor speed reaching at synchronism a value sensibly equal to the magnetizing current M in the primary winding.

The action of the single-phase induction motor running at speed n is as follows, magnetic leakage and primary resistance being ignored: the primary pressure acting on the primary winding produces a pulsating flux Φ, Fig. 3, which produces in the primary winding a pulsational pressure sufficient to balance the impressed pressure. This flux Φ produces in winding A, Fig. 3, a rotational pressure A_r, Fig. 4, which produces the magnetizing current I_1, which in turn produces the cross-flux Φ_c, and this cross-flux in its turn produces a pulsational pressure A_p in A which is equal and opposite to A_r, $R\,I_1$ being negligible. This cross-flux Φ_c produces a rotational pressure B_r in B which opposes the pulsational pressure B_p due to Φ but is considerably less in value so that the sum of B_r and B_p is an unbalanced pressure in phase with Φ_c and this unbalanced pressure produces a large current I_2 in B. The effect of this current I_2 is to permit the flow of an equal and opposite load-current I_0 in the primary coil by virtue of which the primary pressure delivers power to the motor. We shall now proceed to develop the equations of power and torque with the view of applying the Heyland diagram.

The total power delivered to the rotor is delivered to circuit B, Fig. 3, and it is equal to the product of I_2^2 into the fictitious resistance $\left(\frac{f^2}{f^2 - n^2}\right) R$. That is:

$$\text{Total power delivered to rotor} = I^2_2 R \left(\frac{f^2}{f^2 - n^2}\right) \qquad (11)$$

Furthermore:

$$\text{Power lost in circuit } B \text{ as heat} = I_2^2 R \qquad (12)$$

The difference between (11) and (12) is the power which is developed by the torque T which is exerted upon rotor circuit B. Therefore this difference must be equal to $2\,\pi\,n\,T$ so that

$$T = \frac{1}{2\pi} I_2^2 R \left(\frac{n}{f^2 - n^2}\right) \qquad (13)$$

This is the equation which will be used in connection with the Heyland diagram, but a better idea of the relation between torque and speed may be obtained by eliminating I_2^2 from (13) using equation (7). This gives:

$$T = \frac{1}{2\pi} \cdot \frac{E_0^2}{R} \cdot \left(\frac{n}{f^2} - \frac{n^3}{f^4} \right) \tag{14}$$

Therefore, if there is no magnetic leakage so that E_0 may be considered as constant, the torque of a single-phase induction motor is proportional to $\left(n - \frac{n^3}{f^2}\right)$, and the torque is zero at standstill [$n = o$], and also at synchronous speed [$n = f$.]

The cross magnetizing current I_1 in so far as it represents loss of power gets its power from the revolving rotor through the action of a back torque, thus lessening the effective torque of the motor or with given motor load causing a slight decrease in speed and a corresponding slight increase in I_2 and in the primary load current I_0 which is in phase with E_0. The power loss represented by I_1 is partly $I_1^2 R$. The eddy current and hysteresis losses are distributed between the primary magnetizing current M and the cross magnetizing current I_1 approximately as follows: at standstill all the core loss is supplied by the power component of M and at standstill the core loss is large inasmuch as the rotor iron is then the seat of considerable loss. With increasing speed the total eddy current and hysteresis losses decrease and are shared between M and I_1, the part supplied by I_1 being proportional to the speed and equal to half the whole at synchronous speed. Therefore the power component of I_1 increases with increasing speed while the power component of M decreases with increasing speed, but the increase of power component of I_1 with speed means an increase of load current I_0 which supplies to the rotor the mechanical energy which is delivered to I_1 by back torque due to I_1, and one may think of this slight increase of I_0 as compensating the slight decrease of the power component of M so that it is permissible to consider the whole of the core losses as supplied by a constant or nearly constant primary magnetizing current, in which case the loss $I_1^2 R$ is the only loss to be associated with the cross magnetizing current I_1.

The cross magnetizing current I_1 in so far as it is wattless causes a slight pulsating increase and decrease of speed as it takes energy from and gives it back to the revolving rotor. If the moment of inertia of the rotor is large this pulsation of speed is exceedingly small and in this case the wattless component of I_1 has no effect whatever upon the primary current. It is to be particularly noted that time lag of I_1 behind the rotational pressure due to Φ which produces I_1 does not involve any space lag so that the magnetomotive force due to I_1 has no component which opposes the flux Φ either when the resistance of winding A is negligibly small or when the resistance of winding A is large. In other words the currents produced in the actual rotor windings by the rotational pressure due to Φ are always symmetrically distributed with respect to Φ (no space lag) although the maximum value of these currents may occur later than the instant at which $\frac{d\phi}{dt}$ is equal to zero. The idea of the carrying of the rotor currents around by the moving rotor out of the position in which they would be at standstill thus creating a cross magnetomotive force is entirely foreign to the fundamental idea of the magnetic representation of the rotor windings by the stationary windings $A\ A'$ and $B\ B'$; in fact the writer believes that this idea of the production of cross magnetomotive force by the carrying of rotor currents around out of opposition to the primary is essentially a wrong idea. The cross magnetizing current I_1 is produced through the mediation of a rotational pressure which is produced in the rotor conductors as they cut the flux Φ as above explained.

The wattless component of the primary magnetizing current represents a flow of energy from the source of supply to the magnetic field of the motor and back again, while the wattless component of I_1 represents a flow of energy from the kinetic store of the rotor to the magnetic field of the motor and back again. When a two-phase induction motor has one of its stator windings detached from the source of supply, thus becoming a single-phase motor, the wattless component of the magnetizing current in the remaining stator winding is unchanged in value, the power component of this magnetizing current is doubled, and the rotor, before rotating at constant speed, now rotates at a speed which rises and falls slightly at a frequency $2f$. This rise and fall of speed at zero load is due chiefly to the above-mentioned exchange of energy between rotor and cross-flux through the mediation of the wattless component of I_1, while under load the pulsations of speed are due not only to this cause but also to the fact that the load torque is steady while the driving torque, equation (13), pulsates between zero and $2\,T$.

The applicability of the Heyland diagram depends, as stated above, upon the fact that the flow of energy between the primary winding and the B winding of the rotor is identical to the flow of energy by ordinary transformer action from the primary winding to the B winding on the assumption that the B winding has its resistance increased to the fictitious value $\left(\frac{f^2}{f^2-n^2}\right)R$. The ratio of transformation being 1:1 the equivalent resistance of the primary circuit is $R' + \left(\frac{f^2}{f^2-n^2}\right)R$. This resistance varies with speed. The reactance x of the primary is equal to $2\pi f$ times the inductance value of magnetic leakage which may be easily calculated from the dimensions of the motor. This reactance is constant. Therefore the locus of I_0 is a circle of which the diameter is equal to E'/x and at right angles to E', and the locus of total primary current $M+I_0$ is the same circle with the origin displaced as shown in Fig. 5.

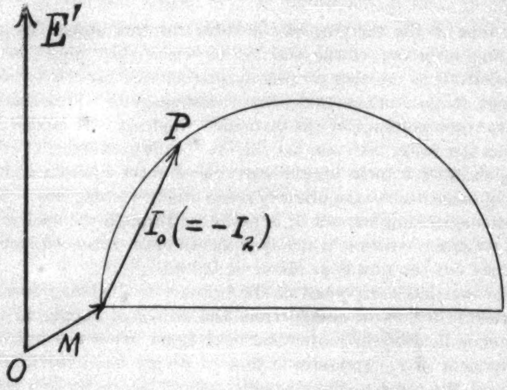

FIG. 5.

Having calculated M and x the diagram Fig. 5 is completely determined. To calculate a set of corresponding values of speed torque, etc., proceed as follows: choose any point P Fig. 5. This determines at once I_2, total primary current $M+I_0$, power-factor, and total power intake. Subtract from the power represented by the load current I_0 the resistance loss in the primary; namely, $(M+I_0)^2 R'$, and we have the total power delivered to the B circuit of the rotor. Thus from equation (11) the speed n may be calculated, I_2, R, and f being known. Knowing n, the total torque exerted on the B circuit may be calculated from equation (13) from which total torque the amount $I_1^2 R/2\pi n$ is to be subtracted, this being the back torque due to rotor circuit A. To calculate this back torque consider that I_1 is sensibly equal to the wattless component of the primary magnetizing current M. The total power output of the motor, ignoring friction, may now be calculated from speed and net torque. Then the efficiency may be calculated. Proceeding in this way for a series of points P Fig. 5 the data for plotting the complete performance curves of the motor may be obtained.

The author's thanks are due to Mr. S. S. Seyfert for a careful verification of the equations given in this paper.

BY-LAWS
OF THE
AMERICAN INSTITUTE OF ELECTRICAL ENGINEERS.

Adopted by the Board of Directors, April 22, 1904.

The Constitution as amended May 21, 1901, authorizes the adoption of By-Laws as follows:

ART. VII. Sec. 4.—*The Board of Directors shall prepare and adopt a series of By-laws which shall govern its procedure and that of its committees, under this Constitution. Such By-laws shall be adopted or may be amended by a two-thirds vote of a quorum of the Board of Directors; provided that the text of a proposed by-law or amendment shall be furnished to each member of the Board of Directors at least ten days before the meeting at which a vote of the same shall be taken.*

BY-LAWS.

MEMBERSHIP.

SEC. 1. Members, Associates and Honorary Members are authorized to use the following abbreviations indicating their grade of membership in the INSTITUTE:

Member,	Mem. A. I. E. E.
Associate,	Assoc. A. I. E. E.
Honorary,	Hon. Mem. A. I. E. E.

It shall be the duty of all Members and Associates to call the attention of the Secretary to the improper use of INSTITUTE badges or symbols, or to claims made by non-members that they belong to the INSTITUTE either as Member or Associate. The Secretary shall thereupon investigate the facts and lay the matter before the Board of Directors.

SEC. 2.—An application for admission as an Associate shall be made upon a printed form approved by the Board of Examiners, and shall embody such information as may be required by the Board to enable it to identify the applicant, and verify his record of experience. The name of the applicant may be posted at the discretion of the Secretary immediately after being filed.

SEC. 3. An applicant for admission as an Associate who is unknown to any Member or Associate of the INSTITUTE, may refer preferably to a member of any grade in the societies designated in Section 5, or to any citizen of standing in the community, and the members of the Board of Examiners may then if satisfied of his merits, endorse his application in the usual manner.

SEC. 4. The Board of Examiners may decline the acceptance of an applicant, although his application be properly endorsed, if for any reason, in the opinion of the Board, the INSTITUTE would not be benefited by such acceptance.

SEC. 5. An applicant for transfer who certifies that he is not personally known to five members, may refer to professional engineers of recognized standing; to an Associate of the AMERICAN INSTITUTE OF ELECTRICAL ENGINEERS; or to an engineer of other branches of engineering, preferably those who are Members of the following named societies: American Society of Civil Engineers; American Society of Mechanical Engineers; American Institute of Mining Engineers; The Canadian Society of Civil Engineers; The Society of Naval Architects and Marine Engineers, all of America; or the Institution of Civil Engineers, the Institution of Mechanical Engineers or the Institution of Electrical Engineers, of Great Britain, or to any other approved American or foreign national engineering body.

SEC. 6. In the absence of replies from applicants to inquiries for information, the Board of Examiners will hold the application in abeyance.

SEC. 7. The names of applicants for transfer to membership, after being recommended for transfer by the Board of Examiners, shall be posted in the PROCEEDINGS and final action thereon shall not be taken by the Board of Directors until one month after such posting.

SEC. 8. Objection to the admission of any candidate for the grade of Associate or for transfer to Membership must be accompanied by reasons for such objection.

SEC. 9. The privileges attaching to membership in the INSTITUTE shall not be accorded to applicants for admission until they have been duly elected and have paid their entrance fees and current dues.

SEC. 10. An Associate elected between the dates of May 1st and November 1st, in any year, shall be entitled to receive the bound volume or volumes of the TRANSACTIONS for the corresponding calendar year, and such single copies of the monthly edition for the same year that he may apply for and which are available.

SEC. 11. An Associate elected between November 1st and May 1st, shall be entitled only to such copies of the PROCEEDINGS as may be published after his election. In case semiannual volumes are issued, he shall be entitled to that volume only which covers the latter part of the calendar year.

SEC. 12. A candidate elected after February 1st and prior to May 1st may at his request be permitted to waive his privilege of receiving the bound volume and monthly PROCEEDINGS and in case of such waiver, the Secretary shall date his annual dues from May 1st, the beginning of the next fiscal year.

SEC. 13 A Member or Associate who has resigned in good standing, may on the approval of the Board of Examiners and the Board of Directors be reinstated without payment of entrance fee, and be entitled to admission to the grade originally held by him. If the applicant so elects, he may retain his original date of election and transfer by payment of all dues which would have accrued during the interim, and he will then be entitled to such INSTITUTE publications issued during such period, as are available.

SEC. 14. The rights and privileges of each Honorary Member, Associate, or Student, shall be personal to himself, and shall not be transferable or transmissible by his own act or by operation of law.

SEC. 15. A Member or Associate resigning from the INSTITUTE or permitting his membership to lapse through non-payment of dues, has no claims to proprietary interest in the property of the INSTITUTE.

SEC. 16. A Member or Associate desiring to resign, who may be in arrears for dues, may return, carriage paid, his bound volumes of the TRANSACTIONS, if in good condition, and be given credit therefor at $2.50 each; the amount to apply only in settlement of his account, and not to be paid for in cash.

PUBLICATIONS.

SEC. 17. The published lists of Members and Associates shall be issued annually in the form of a Catalogue, uniform with the TRANSACTIONS in size of page, with names of Members and Associates properly classified and arranged alphabetically; also annually in the form of a Directory, in which the names of Members and Associates are distinguished by Italic and Roman type respectively, and arranged geographically.

SEC. 18. The regular publications of the INSTITUTE shall consist of PROCEEDINGS, to be published monthly, and TRANSACTIONS, to be published semi-annually or annually.

SEC. 19. The PROCEEDINGS shall be issued regularly on the first day of each calendar month, and shall have two principal divisions—the first for current matter and INSTITUTE news, the second for technical papers and discussions. In one part shall be included matter of temporary interest, such as announcements, minutes of the Board, election of members, and general matter pertaining to the INSTITUTE. Technical papers, the discussions thereof, and matters of an engineering character shall be included in the other part.

SEC. 20. The TRANSACTIONS of the INSTITUTE shall be published annually in one or more volumes, as may be determined from time to time by The Editing Committee, and shall consist of the papers, discussions and other engineering matter, as printed in the PROCEEDINGS, properly rearranged and repaged.

SEC. 21. The paper or papers to be presented at the meeting in any month shall be published in the PROCEEDINGS in advance of the meeting. The papers may be published in any previous number of the PROCEEDINGS, but must be published, at the latest, in the PROCEEDINGS for the month in which the meeting is held.

SEC. 22. All papers, discussions and other matter, intended for publication in the PROCEEDINGS, shall be edited and revised in the manuscript.

SEC. 23. The manuscript of a paper or discussion, after being edited and revised for publication under the direction of The Editing Committee, shall be returned to the author to obtain his consent to publication as revised. In case the author shall not consent to publication in the form suggested by The Editing Committee, the paper or discussion shall not be published.

SEC. 24. No manuscript of a paper or written contribution to a discussion shall be given to the press for publication in advance of its appearance in the PROCEEDINGS, unless by authority of the Board of Directors or of The Editing Committee.

SEC. 25. The acceptance of a paper by the Committee on Papers shall not be considered a guarantee of its publication.

SEC. 26. The INSTITUTE shall publish annually a year-book containing a catalogue of the membership, and the Constitution and By-Laws of the INSTITUTE. If expedient, this year-book may form one of the numbers of the PROCEEDINGS; otherwise, it shall be published separately.

SEC. 27. The INSTITUTE may publish a catalogue of the library as part of one of the numbers of the PROCEEDINGS, or as a separate issue, as may seem expedient.

SEC. 28. The Secretary is authorized to receive annual subscriptions for the monthly PROCEEDINGS, ending with the calendar year, at the rate of $5.00 per annum for all countries to which the bulk rate of postage applies. To all other countries the price shall be $6.00 per year, beginning January 1, 1905.

SEC. 29. Existing bound volumes, or bound volumes hereafter published may be sold to non-members at the following prices:

Paper covers	(Single Volumes)	$5.00
Cloth	do.	5.75
Half-morocco	do.	6.50

The price of volumes in paper only, prior to Vol. V., shall be $2.00 each. A discount of 10 per cent. from the above prices, either for volumes, subscriptions or parts, may be allowed to publishers and dealers. Members and Associates shall be allowed a discount of 50 per cent. on parts or duplicate volumes to replace those lost or destroyed. They may purchase volumes issued prior to their election at $5.00 each either in paper or cloth covers or $1.00 extra for half morocco binding.

BOARD OF DIRECTORS.

SEC. 30. A motion to rescind a resolution adopted by the Board of Directors may be entertained at its next regular meeting, provided the Secretary shall mail to each member of the Board of Directors a copy of such notice to rescind, at least five days before the next regular meeting is held.

Sec. 31. Special meetings of the Board of Directors or of the Executive Committee may be held at the call of the President. Notice of such special meeting shall be mailed to each member of the Board of Directors or of the Executive Committee at least five days before the time appointed for holding such meeting; such notice to contain a synopsis of the business to be brought before such special meeting. No business other than that specified in the notice for a special meeting shall be transacted at such meeting. A waiver of notice, signed by all the members of said Board of Directors or Executive Committee may be allowed in lieu of such notice.

Sec. 32. Regular meetings of the Board of Directors shall be held each month on the afternoon of the regular monthly meeting of the Institute, at such hour as may be fixed by the President and Secretary.

Sec. 33. The expenditures of the Institute shall be regulated by annual appropriations which shall begin on the first day of November of each year. Each committee shall prepare and submit to the Board of Directors at its regular October meeting an estimate of its requirements for the ensuing year, and the Board of Directors shall consider and act on these estimates

Sec. 34. A quorum of the Board of Directors shall consist of five members for all business, except the purchase, sale, lease or mortgage of real estate and the approval of the Annual Report. For any business involving the purchase, sale, lease or mortgage of real estate and for the approval of the Annual Report, a majority of the Board of Directors shall constitute a quorum.

Committees and Duties.

Sec. 35. Each standing committee shall have a regular monthly day of meeting, as far as practicable. Each committee shall keep a record of its proceedings, including the attendance of its members. All resolutions or reports which each committee shall forward to the Board of Directors for action, must be signed by the Chairman, and a majority of its members.

Sec. 36. It shall be the duty of the Committee on Finance to audit all bills before payment; to make recommendations to the Board of Directors respecting the proper investment or disposition of the funds of the Institute; and to direct the attention of the Board to any charges whatever which it may consider excessive or improper. It shall also prepare for the information of the Board of Directors a balance sheet each year to October 31st, showing the financial condition of the Institute, including the state of each fund which may be separately invested. It shall also be the duty of the Committee on Finance to examine the bank books and securities of the Institute prior to the annual financial statement made in the month of May, and to certify to their correctness.

Sec. 37. It shall be the duty of the Library Committee to be responsible for the care and maintenance of the library; to employ a competent librarian; and to accept donations of books or money for library purposes, on behalf of the INSTITUTE. It shall prepare an annual report up to the first of May, each year, showing the state of the library, and giving the names of all donors to the library for the preceding year, which report, when accepted by the Board of Directors, shall be printed, and form part of the monthly PROCEEDINGS All moneys received by the Library Committee or any of its members, which has been contributed for library purposes, shall be turned over to the Treasurer and paid out by him for the purpose specified by resolution of the Board of Directors. The Library Committee may exchange copies of INSTITUTE PROCEEDINGS or TRANSACTIONS, if available, for periodicals or other publications.

SEC. 38. It shall be the duty of the Committee on Papers to arrange for all regular and special meetings of the INSTITUTE at which papers are presented. It shall solicit and procure from Members and Associates suitable papers to be presented at meetings, which papers may afterwards be printed in the PROCEEDINGS and TRANSACTIONS. It shall not accept any paper from a non-member unless directly authorized to do so by a resolution of the Board of Directors.

SEC. 39. The Editing Committee shall prepare the papers and discussions for publication, and shall exercise a general supervision over all publications of the INSTITUTE. It shall preferably publish all discussions in the third person, and may revise, condense or omit any part of such discussions. It shall also call the attention of the Board of Directors to papers which it may deem unsuitable for publication in the PROCEEDINGS or TRANSACTIONS, assigning specific reasons therefor.

SEC. 40. It shall be the duty of the Board of Examiners to carry out strictly the provisions of the Constitution respecting the election of Associates and the transfer of Associates to full membership. In judging the qualifications of Associates who have applied for transfer, the Board of Examiners shall be authorized to pass upon the standing of any educational institution. No application for transfer shall be considered by the Board of Examiners until after the applicant has been elected and qualified as an Associate. The Board of Examiners is authorized to pass upon the competency of references of applicants for election as Associates, and for transfer to the grade of Member.

SEC. 41. No standing or temporary committee under any circumstances shall exceed its authorized appropriation without first obtaining the sanction of the Board of Directors.

SEC. 42. Temporary committees appointed to arrange for the holding of the Annual Convention, or for any other purpose involving expense to the INSTITUTE, shall in all cases prepare an estimate of the probable expenditure, and submit it to the Board of Directors for its approval, and the sum granted by the Board shall be in no case exceeded without the previous assent of the Board to any additional expenditure in excess of the original estimate.

SEC. 43. The functions and duties of each temporary committee shall be defined by a resolution of the Board of Directors. At the beginning of each administrative year the Secretary shall transmit to all the members of each temporary committee whose duties may be continued, a copy of the original resolution constituting such temporary committee. It shall be the duty of the Secretary to call the attention of the Board of Directors to work done by any temporary committee outside the scope of the resolution or resolutions which constituted it.

MEETINGS.

SEC. 44. The announcements of fixed dates of meetings, distributed by mail at any date prior to the ten days constitutional requirement, shall be deemed a valid notice of such meetings.

SEC. 45. The President or acting President at any regular or special meeting of the INSTITUTE or the Chairman or acting Chairman at any Branch meeting, may invite any non-member present to discuss any paper; and the Board of Directors may direct The Editing Committee to publish such discussion subject to the rules governing the publication of discussion by members, provided that his remarks are germane to the subject under discussion, and are of sufficient merit to justify an exception to the general rule that discussions by non-members shall not be printed in the TRANSACTIONS.

BRANCHES AND BRANCH MEETINGS.

SEC. 46. Branches of the INSTITUTE may be formed in order to extend the benefits of its membership and to increase its influence throughout the United States and foreign countries.

SEC. 47. A Branch should be the center of local activity in electrical engineering matters, and should aim to bring together electrical engineers and others interested in electricity. The principal object of a Branch shall be the discussion of the technical papers and other proceedings of the meetings of the INSTITUTE; other papers, however, may be presented and discussed at meetings of a Branch, preferably papers relating to the subjects then under consideration at the regular meetings; but other engineering matters may be made the subject of meetings.

SEC. 48. A Branch of the INSTITUTE may coöperate with other local engineering organizations in the holding of joint meetings, and may invite members of such organizations to the meetings of a Branch.

SEC. 49. Upon the statement in writing of a Member or Associate that at least ten Members and Associates of the INSTITUTE, residing in one locality, desire to organize as a Branch of the INSTITUTE, the Board of Directors may authorize the formation of a Branch for that locality, to be known as the [Name of the place, or University] Branch of the AMERICAN INSTITUTE OF ELECTRICAL ENGINEERS. After such authorization the Members and Associates uniting to form the Branch shall organize by the election of an Executive Committee of not fewer than three members, and this Committee shall elect from its members, a Chairman, a Local Secretary, and such other officers as it may desire. This organization shall be reported to the Board of Directors, with the names of the officers elected. The Board of Directors reserves the right to veto the election of any member of the Committee, or of any officer, and to call for the election of another in his place. Upon the approval of the preliminary organization by the Board of Directors, such organization shall be considered a permanent Branch of the INSTITUTE.

SEC. 50. The general control of a Branch shall be vested in the Board of Directors. The Board of Directors may at any time rescind the authorization of any Branch and terminate its existence.

SEC. 51. It shall be the duty of the Chairman to foster the growth of the INSTITUTE, through the Branch, and for this purpose he shall, among other things, endeavor to procure papers of value, and to promote sound discussion at the meetings.

SEC. 52. It shall be the duty of the Local Secretary of a Branch to report its proceedings to the Secretary of the INSTITUTE, to preside at meetings from which the Chairman may be absent, and, in general, to discharge the duties of a Secretary, both to the Branch itself, and in its relation to the INSTITUTE.

SEC. 53. All papers and discussions thereof presented at the meeting of a Branch shall be the property of the INSTITUTE, and in general shall not be published in the PROCEEDINGS. They shall be sent by the Local Secretary to the Secretary of the INSTITUTE. If duly approved, the paper may then be printed in the technical press, with the statement that it has been presented before a Branch. If not approved for publication, to be returned to the author; and if such unapproved paper is then published by the author it shall not be accompanied by the statement that the paper has been presented at a meeting of a Branch, nor shall such fact be advertised in any way in the publication of the paper. Papers read at a Branch and discussions thereon that are submitted to the Secretary, will be referred to The Editing Committee for action regarding publication in the PROCEEDINGS and TRANSACTIONS. A paper may be accepted by a Branch from a non-member, and may be sent to the Secretary of the INSTITUTE, as above prescribed, for approval for publication in the technical press; in general, however, a paper by a non-member shall not appear in the PROCEEDINGS or TRANSACTIONS.

SEC. 54. The INSTITUTE will pay the reasonable expenses of the meetings of a Branch, or such portion thereof as it may see fit, upon the presentation of original bills, properly endorsed by the Local Secretary of the Branch. Each Branch shall submit to the Committee on Finance in advance, an estimate of its proposed expenses for the year ending October 31st.

STUDENTS.

SEC. 55. A person pursuing a regular course of study in electrical subjects, may be enrolled as a " Student of the AMERICAN INSTITUTE OF ELECTRICAL ENGINEERS," as herein provided.

SEC. 56. A person desiring to be enrolled as a student shall make a written application upon a form provided for that purpose, which application shall be endorsed by a Member or Associate of the INSTITUTE.

SEC. 57. A fee of $3.00, covering one year, must be paid in advance for this privilege. Upon receipt of this fee, the applicant's name may be placed upon the mailing list, subject to the approval of the Board of Examiners and the Board of Directors. The privileges of enrolment as a Student may be continued for three years, provided that at the expiration of each year the fee of $3.00 for the following year be paid within 30 days after presentation of the bill for that year.

SEC. 58. Applicants who have received engineering degrees are not eligible for enrolment as Students, unless their applications were filed before graduation.

SEC. 59. A person who is regularly pursuing electrical studies, but who is not a graduate of a technical college or school, may be enrolled as a Student as herein provided.

SEC. 60. Students enrolled under the provisions of these By-laws, shall each be entitled to receive regular monthly PROCEEDINGS and ordinary notices of meetings. They will be permitted to purchase single bound volumes of the TRANSACTIONS at $3.50 each.

SEC. 61. These privileges may be withdrawn from any Student at any time by a vote of the Board of Directors.

SEC. 62. A Student changing his mailing address without notifying the Secretary, shall not be entitled to duplicate copies of the PROCEEDINGS except upon payment of 25 cents for each copy furnished.

STUDENT-MEETNIGS.

SEC. 63. Any Member or Associate who may be connected with a college or engineering school of recognized standing, having a course in electrical engineering, may, upon the approval of the Board of Directors, organize Student-Meetings, to be held under his direction. STUDENT-MEETINGS in institutions in which there are ten or more Students, of the INSTITUTE may be designated as [name of institution] Branch of the AMERICAN INSTITUTE OF ELECTRICAL ENGINEERS.

SEC. 64. Student-Meetings shall organize by the appointment or election of an Executive Committee of any suitable number of which the said Member or Associate shall be a member, and of which other Members or Associates or the enrolled Students of the INSTITUTE may constitute the other members. This organization shall be subject to the approval of the Board of Directors. Other students in the electrical engineering courses of the institutions may attend such Student-Meetings, under conditions to be prescribed by its Secretary.

SEC. 65. The Member or Associate by whom the Student-Meetings shall be organized shall be responsible to the Secretary of the INSTITUTE for the conduct of such Student-Meetings, and shall make regular reports to the Secretary of the proceedings at such meetings.

SEC. 66. The principal object of Student-Meetings shall be the discussion of papers read before the regular meetings of the INSTITUTE. The Secretary of the INSTITUTE may send to Student-Meetings a limited number of advance copies of the papers of the INSTITUTE, in addition to those to be sent regularly to the enrolled Students of the INSTITUTE.

SEC. 67. The INSTITUTE will publish from time to time a list of colleges and schools in which Student-Meetings are held, and brief notices of the proceedings of such Student-Meetings.

JOHN FRITZ MEDAL.

SEC. 68. The representatives of the INSTITUTE on the Board of Award of the John Fritz Medal, shall be the President of the INSTITUTE for the time being and his three immediate predecessors in office. Each year the president-elect after assuming the active duties of his office shall on the third Friday in January become a representative of the INSTITUTE on the Board of Award of the John Fritz Medal and the functions of the senior past-president in connection with this award, shall automatically cease and determine. [a]In the event of the death or permanent disability of any of the representatives of the INSTITUTE on the Board of Award of the John Fritz Medal, the Board of Directors shall be empowered to select his successor from among the surviving past-presidents of the INSTITUTE and such successor shall hold office for the term of the representative in whose place he is appointed.

APPOINTMENTS, NOMINATIONS AND ELECTIONS.

SEC. 69. The election of a Vice-president to the office of President shall create a vacancy to be filled by the Senior Manager as provided by the Constitution; and the Board of Directors shall fill the vacancy thus caused by the appointment of a Manager selected from the remaining candidates whose names appeared on the General Proposal List prepared for the last annual election.

SEC. 70. In preparing the Directors' Ticket, the names proposed after being seconded, shall be voted for by written ballot. The names of candidates for Vice-president should be selected as far as practicable from different sections of the country.

SEC. 71. It shall be the duty of the Secretary to mail or deliver to the membership the nomination forms, envelopes and membership lists on the first or second day of February of each year, and he may at his discretion send such matter to distant points at first-class postal rates.

SEC. 72. Nomination forms as provided shall be sent upon application to Associates who have qualified between the dates of February 1st and March 1st, and single copies only of such nomination forms shall be sent upon request as duplicates to Members or Associates who may have mislaid or destroyed their original copies. Nominations may, however, be written on ordinary letter paper if the regular form is not available.

SEC. 73. The President shall appoint two Tellers to assist the Secretary in canvassing the nominations, and that duty shall be performed not later than March 15th of each year.

SEC. 74. Should the President for any reason fail to appoint five Tellers at the April meeting of the Board of Directors, he may appoint them by letter thereafter as provided in Article VI., Section 5, of the Constitution.

SEC. 75. The Chairman of the Committee on Tellers shall execute a duplicate copy of the report provided for in Article VI., Section 7, of the Constitution, which he shall seal and deposit in the safe at the INSTITUTE headquarters.

PAYMENT OF DUES.

SEC. 76. Annual dues are payable in advance upon receipt of bills dated May 1st, the beginning of each fiscal year.

SEC. 77. No bound volumes shall be sent to Members or Associates whose dues for the current or preceding years remain unpaid.

SEC. 78. No monthly issues of the PROCEEDINGS shall be sent to Members or Associates who shall be in arrears for one year in addition to the dues of the current year.

SEC. 79. Members or Associates who are in arrears for two years in addition to the current year, shall have their names placed upon the suspended list, and no dues shall be permitted to accrue thereafter.

SEC. 80. During the third year of arrearage, such suspended members shall be notified that their names have been placed upon the suspended list, and that unless some suitable explanation of their default is given, their names will be presented to the Board of Directors for removal from the roll of membership of the INSTITUTE.

SEC. 81. Statements of account shall be sent to each Member or Associate whose dues remain unpaid for the quarters beginning August 1st, November 1st and February 1st.

SEC. 82. A printed slip embodying the By-laws covering the payment of dues shall be enclosed by the Secretary at his discretion, whenever bills or statements are distributed.

SEC. 83. The mailing of such bills or statements to the last address upon the office Mailing List, shall be considered a valid notice of indebtedness.

SEC. 84. The name of any Member or Associate in arrears for more than one year, prior to the current year, may be omitted from the Mailing List and Directory.

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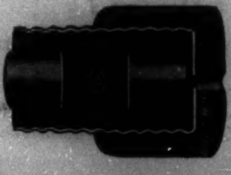

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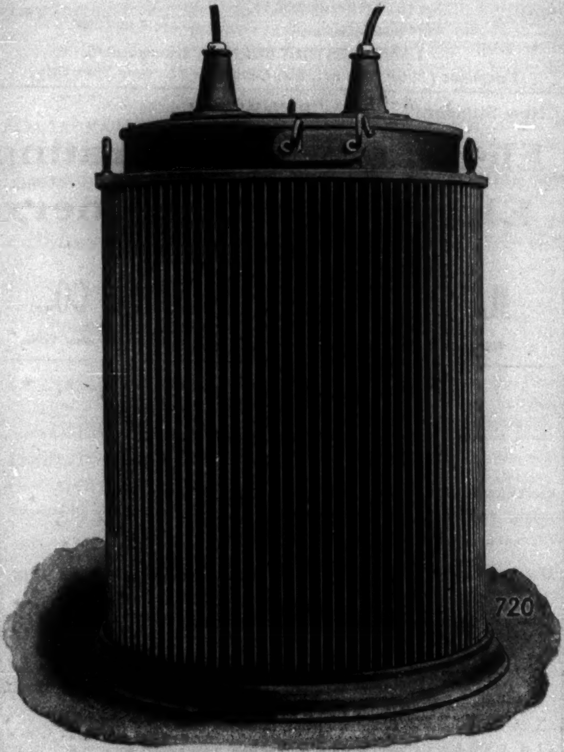

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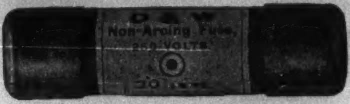

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